Reg. No. : ................................

Name : ....................................



**K18U 2258**

**V Semester B.Sc. Hon's (Mathematics) Degree (Regular) Examination, November 2018**
**BHM 501 : SPECIAL FUNCTIONS**
**(2016 Admission)**

Time : 3 Hours                                                                 Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define a power series.
2. When do you say a singular point of a differential equation to be regular ?
3. Give an example of a differential equation with an ordinary point at $x = 0$.
4. Write down the $n^{th}$ Legendre Polynomial $P_n(x)$.
5. Write down the Bessel function of the first kind of order p. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Define a singular point of a second order linear homogeneous differential equation and give an example of a differential equation with a singular point.
7. For the differential equation $x^3(x - 1)\ y'' - 2(x - 1)y' + 3xy = 0$, locate and classify its singular points on the x-axis.
8. Determine the nature of the point $x = 0$ for the differential equation $x^2 y'' + (\sin x)y = 0$.
9. Write down the Gauss's hypergeometric differential equation and test the nature of its singular points.

P.T.O.

10. Verify that $(1 + x)^p = F(-p, b, b, -x)$ by examining the series expansions of the functions on the left sides.

11. State the orthogonality property of Bessel functions.

12. Write short notes on Legendre series.

13. Prove that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

14. Define an improper integral and classify them. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. Find a power series solution of the form $\sum_{n=0}^{\infty} a_n x^n$ to solve the differential equation $y' + y = 1$ and verify your conclusion by solving the equation directly.

16. Check the nature of the point $x = 0$ for the differential equation $y'' + y' - xy = 0$ and find a power series solution for it which satisfy the conditions $y_1(0) = 1$, $y'_1(0) = 0$.

17. Find the indicial equation and its roots for the differential equation $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$.

18. Write down the hypergeometric series F(a, b, c, x) and hence verify that the following by examining the series expansions of the functions on the left sides :

    i) $\sin^{-1}x = xF\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2\right)$

    ii) $e^x = \lim_{b\to\infty} F\left(a, b, a, \frac{x}{b}\right)$

19. Find the general solution of $(x^2 - 1)y'' + (4 + 5x)y' + 4y = 0$ near the singular point $x = -1$, in terms of Gauss's hypergeometric series.

20. Determine the nature of the point $x = \infty$ for the differential equation $y'' + \frac{4}{x}y' + \frac{2}{x^2}y = 0$.

21. Derive Rodrigues' formula.

22. Find the first three terms of the Legendre series of $f(x) = \begin{cases} 0, & \text{if } -1 \le x \le 0, \\ x, & \text{if } 0 \le x \le 1, \end{cases}$



23. Prove that (i) $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$ and (ii) $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$.

24. Classify according to the type of improper integral :

    i) $\int_{-1}^{1} \frac{1}{\sqrt[3]{x}\,(x+1)}\,dx$

    ii) $\int_{3}^{10} \frac{x}{(x-2)^3}\,dx$

    iii) $\int_{-\infty}^{\infty} \frac{x^2}{x^4 + x^2 + 1}\,dx$

25. Investigate the convergence of the following integrals :

    i) $\int_{1}^{5} \frac{1}{\sqrt{(5-x)(x-1)}}\,dx$

    ii) $\int_{0}^{\pi} \frac{\sin x}{x^3}\,dx$

    iii) $\int_{2}^{3} \frac{1}{x^2(x^3+8)^{2/3}}\,dx$

26. Prove that Beta function is symmetric and $B(m, n) = 2\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta\, d\theta$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Find two independent Frobenius series solutions for the differential equation $4xy'' + 2y' + y = 0$.

28. Determine the nature of the point $x = \infty$ for the Legendre's equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$, and find the exponents from the indicial equation.

29. State and prove the orthogonality property of Legendre polynomials.

30. Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, m, n > 0. **(2×6=12)**