SECTION – D

Answer **any one** question. **Each** question carries **10** marks. **(1×10=10)**

34. State and prove Cauchy's integral formula.

35. If a function f is analytic throughout an annular domain $R_1 < |z - z_0| < R_2$, centred at z_0 and C denote the positively oriented simple closed contour around z_0 and lying in that domain, prove that f(z) has a series expansion.

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}, \quad R_1 < |z - z_0| < R_2,$$

where $a_n = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{n+1}} dz \quad (n = 1, 2, 3, \ldots)$

and $b_n = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{-n+1}} dz \, (n = 1, 2, 3 \ldots)$

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, November 2018
BHM 504 : COMPLEX ANALYSIS – I
(2013 – 15 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark. **(10×1=10)**

1. Write the function $f(z) = z^2$ in the form $f(z) = u(r, \theta) + iv(r, \theta)$.
2. Find the derivative of $(2z^2 + i)^5$.
3. Define entire functions and give an example.
4. Prove that $|\cosh z|^2 = \sinh^2 x + \cos^2 y$.
5. Evaluate $\int^{\frac{\pi}{6}} e^{izt}\, dt$.
6. By finding an antiderivative, evaluate $\int_0^{\pi+2i} \cos(z/2)\, dz$, where the path is any contour between 0 and $\pi + 2i$.
7. If C is any closed contour lying in the open-disk $|z| < 2$, find the value of $\int_C (ze^z) / (z^2 + 9)^5 \, dz$.
8. What is the image of each branch of a hyperbola $x^2 - y^2 = c$, $c > 0$, under the mapping $w = z^2$?
9. Examine whether $T(x, y) = e^{-y}. \sin x$ is harmonic.
10. Find the zeros of sinhz.

P.T.O.

SECTION – B

Answer **any 10** questions. **Each** question carries **3** marks. **(10×3=30)**

11. Write $f(z) = x^2 - y^2 - 2y + i\,(2x - 2xy)$ in terms of z, where $z = x + iy$.

12. Prove that the real and imaginary parts of $f(z) = e^z$ satisfy the Cauchy – Riemann equations.

13. If f(z) is analytic in a given domain D and if |f(z)| is a constant throughout D, show that f(z) is a constant.

14. Prove that $\cos^{-1} z = -i \log\left[z + i\left(1 - z^2\right)^{1/2}\right]$.

15. Find $\int_C f(z)\,dz$, where $f(z) = (z + 2)/z$ and C is the semi circle $z = 2e^{i\theta}$, $0 \le \theta \le \pi$.

16. If C is a positively oriented unit circle $|z| = 1$, evaluate $\int_C \frac{\exp(2z)}{z^4}\,dz$.

17. If C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $b = \pm 2$, find $\int_C \frac{z}{2z+1}\,dz$.

18. Find the harmonic conjugate of $u(x, y) = y^3 - 3x^2y$.

19. Find the analytic function $f(z) = u + iv$ of which the real part is $u = e^{-x}[(x^2 - y^2)\cos y + 2x \sin y]$.

20. Prove that the function $f(z) = x^2y^5(x + iy)/(x^4 + y^{10})$, $z \ne 0$ and $f(0) = 0$ satisfy the Cauchy – Riemann equations at the origin.

21. Show that the function $f(z) = xy + iy$ is not analytic.

22. Obtain the Maclaurin's series representation of $f(z) = \frac{1}{1-z}$.

23. Expand $f(z) = \frac{1}{z-1} - \frac{1}{z-2}$ into a series involving powers of z for $|z| < 1$.

24. Prove that the power series $S(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n$ can be differentiated term by term.

SECTION – C

Answer **any 6** questions. **Each** question carries **5** marks. **(6×5=30)**

25. Show that the line $x = a$ in the z – plane corresponds to the parabola $v^2 = -4a^2(u - a^2)$ under the mapping $w = z^2$.

26. Let $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$, $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$. If $\lim_{(x,y)\to(x_0,y_0)} u(x, y) = u_0$, $\lim_{(x,y)\to(x_0,y_0)} v(x, y) = v_0$. Show that $\lim_{z\to z_0} f(z) = w_0$.

27. Find the value of $\left|\int_C \bar{z}\,dz\right|$ where C is the right hand half $z = 2e^{i\theta}$, $-\pi/2 \le \theta \le \pi/2$.

28. State and prove the Fundamental theorem of Algebra.

29. Prove that $\int_C z^{a-1}dz = i\frac{2R^a}{a}\sin(a\pi)$, where C is the positively oriented circle. $z = Re^{i\theta}$, $-\pi \le \theta \le \pi$, about the origin and 'a' is any non-zero real-number.

30. If C is a contour of length L, f(z) is piecewise continuous on C and M is a non-negative constant such that $|f(z)| \le M$ for all points z on C at which f(z) is defined, show that $\left|\int_C \bar{z}\,dz\right| \le ML$.

31. If a function f is entire and bounded in the complex planes, prove that f(z) is a constant throughout the plane.

32. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, show that its component functions u and v are harmonic in D.

33. Represent the function $f(z) = \frac{4z+4}{z(z-3)(z+2)}$ in Laurent's series when i) $0 < |z| < 1$ and ii) $2 < |z| < 3$.