K16U 2589

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2016
BHM 504 : COMPLEX ANALYSIS – I

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Define interior point of a set on the complex plane.

2. Find the accumulation point of $\left\{\frac{i^n}{n} = 1, 2, 3, \ldots\right\}$.

3. Find the domain of the function $f(Z) = \frac{1}{z^2+1}$.

4. State true or false. The function $f(Z) = \overline{Z}$ is continuous everywhere but differentiable nowhere.

5. Define a harmonic function.

6. Give an example of an entire function.

7. If $Z_n = -2 + i\frac{(-1)^n}{n^2}$, $n = 1, 2, 3, \ldots$, evaluate $\lim_{n\to\infty} Z_n$.

8. What do you mean by the circle of convergence of a power series ?

9. State Taylor's Theorem.

10. Find the principal value of $\frac{i}{-2-2i}$.

P.T.O.

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. Find all values of $(8i)^{1/3}$.

12. Show that $\lim_{z \to -1} \frac{iz+3}{z+1} = \infty$.

13. Sketch the sets $S_1 = \{z \in C : |2Z+3| > 4\}$ and $S_2 = \{z \in C : \text{Im } Z \leq 1\}$ and determine which are domains.

14. Show that $u(x, y) = y^3 - 3x^2 y$ is harmonic and find a harmonic conjugate $v(x, y)$.

15. Evaluate $\int_C f(z)\, dZ$ where $f(Z) = x^2 + 3ixy$ and C is a line joining $1 + i$ to the point $2 - i$.

16. Apply Cauchy-Goursat theorem to show that $\int_C f(Z)dZ = 0$ where the contour C and f(Z) are given by $f(Z) = \tan Z$ and $C : |Z| = 1$.

17. Find the Laurents series $Z^2 e^{\frac{1}{Z}}$ about 0.

18. Show that the function $f(Z) = Z + \overline{Z}$ is nowhere differentiable.

19. Show that $(\text{Cosh} Z)^2 = \text{Sinh}^2 x + \text{Cos}^2 y$.

20. If $f(Z) = u(x, y) + i\, v(x, y)$ is analytic, show that f(z) must be a constant throughout D if $f'(Z) = 0$ for all $Z \in D$.

21. If $\lim_{z \to z_0} f(Z)$ exists, show that it must be unique.

22. If v is a harmonic conjugate of u in a domain D, then prove that – u is a harmonic conjugate of v in D.

23. Show that $\int_C \frac{1}{z} dZ = 2\pi i$ where C is any positively oriented closed contour surrounding the origin.

24. State and prove Liouvilles theorem.

Answer **any 6** short essay questions out of 9. **(6×5=30)**

25. Show that an analytic function f(Z) is constant through a region D if |f(Z)| is a constant.

26. If a function is analytic, show that it is independent of $\overline{Z}$.

27. Find the Laurents series expansion of $\frac{1}{z(1+z^2)}$ $0 < |Z| < 1$.

28. Integrate the function $f(Z) = \frac{e^z}{(z-1)^2(z^2+4)}$.

29. If f(Z) is a regular function of Z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(Z)|^2 = 4|f'(Z)|^2$.

30. Obtain the formula of inverse sine function $\sin^{-1}(Z)$.

31. Evaluate $\int_C f(Z)dZ$ where $f(Z) = Z^2$ and C is the parabola $y = x^2$ from (0, 0) to (2, 4).

32. Prove that every absolute convergent series of complex numbers is convergent.

33. Show that when $Z \neq 0, \frac{\sinh Z}{z^2} = \frac{1}{z} + \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+3)!}$.

Answer **any one** essay question out of 2. **(1×10=10)**

34. a) Prove that the function $u = 4xy - x^3 + 3xy^2$ is harmonic.

    b) Find the harmonic conjugate of u and find the corresponding analytic function f in terms of Z.

35. a) State and prove Maximum Modulus Principle.

    b) State Cauchy-Goursat theorem.