K15U 0348

Reg. No.: ....

Name : ....

V Semester B.Sc. (Hon's) (Mathematics) Degree (Regular) Examination, November 2015
BHM 504 : COMPLEX ANALYSIS – I

Time : 3 Hours Max. Marks : 80

Answer **all** the questions. **(10×1=10)**

1. Define an accumulation point of a set on the complexplane.
2. Give an example of a set which is neither open nor closed.
3. Find the domain of the function $f(Z) = \frac{1}{1-|z|^2}$.
4. Define a singular point of a function.
5. Define a simply connected domain.
6. Is the function $f(Z) = |Z|^2$ analytic at the origin ?
7. Determine the principal value of argument of $\frac{-2}{1+\sqrt{3}\,i}$.
8. What is the Maclaurin series expansion of e^z ?
9. State Laurent's theorem.
10. State fundamental theorem of algebra.

P.T.O.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Find all the values of $\left(\sqrt{3}+i\right)^{1/2}$.

12. Show that $\lim_{z\to\infty}\dfrac{2z^3-1}{z^2+1}=\infty$.

13. Sketch the set $S=\{Z\in C:|z+1|+|z-1|<4\}$.

14. Prove that $u=x^2-y^2$ and $v=\dfrac{-y}{x^2+y^2}$ are harmonic, but $u+iv$ is not an analytic function of Z.

15. Evaluate $\int_C \bar{Z}\,dz$ where C is the right half of the circle $|Z|=2$ from $Z=-2i$ to $Z=2i$.

16. Apply Cauchy – Goursat theorem to show that $\int_C Z\,dz=0$ where the contour C and f(Z) are given by $f(Z)=\dfrac{z^2}{z^2+9}$ and $C:|Z-1|=1$.

17. Find the Taylor series expansion of the function $f(Z)=\sin Z$ about $Z=\dfrac{\pi}{2}$.

18. Prove that every differentiable function is continuous.

19. Determine whether $\lim_{z\to 0}\dfrac{z}{\bar{z}}$ exists.

20. Show that $(\sinh Z)^2=\sinh^2 x+\sin^2 y$.

21. Find the Laurent's expansion of the function $f(Z)=\dfrac{1}{z^3-z^4}$ at $Z=0$.

22. Prove that the series $\sum_{n=0}^{\infty}\dfrac{(3-4i)^n}{n!}$ converges.

23. Determine the constants a and b such that $f(Z)=(x^2+ay^2-2xy)+i(bx^2-y^2+2xy)$ should be analytic.

24. Define a connected set on the complex plane. Sketch the set $S=\{Z\in C:|\operatorname{Re}Z|>1\}$. Is it a connected set ?

Answer **any 6** short essay questions out of **9**. **(6×5=30)**

25. Show that an analytic function f(Z) is a constant in a domain D if $|f(Z)|$ is a constant in D.

26. If a function $f(Z)=u+iv$ is analytic in a domain D, prove that its component functions u and v are harmonic in D.

27. Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ of it.

28. Obtain the Laurents series expansion of the function $f(Z)=\dfrac{-1}{(z-1)(z-2)}$ valid in the region $1<|Z|<2$.

29. Show that $\dfrac{1}{1-z}=\sum_{n=0}^{\infty}Z^n\ (|Z|<1)$.

30. If $f'(Z)=0$ everywhere in a domain D, show that f(Z) must be a constant throughout D.

31. If Z_0 and W_0 are two points in Z and W plane, show that $\lim_{z\to\infty}f(z)=\infty$ if and only if $\lim_{z\to 0}\dfrac{1}{f\left(\frac{1}{z}\right)}=0$.

32. Integrate the function $f(Z)=\dfrac{z^4-3z^2+6}{(z+i)^3}$ counter clockwise sense around the circle $|Z|=\dfrac{3}{2}$.

33. State and prove Liouville's theorem.

Answer **any one** essay question out of **2**. **(1×10=10)**

34. a) State and prove Cauchy integral formula.

    b) Obtain the extension of Cauchy integral formula.

35. a) Derive the Cauchy Riemann equations in Cartesian form.

    b) State the sufficient condition for differentiability.

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