31. Find the extremals of the functional $S = \int_{x_0}^{x_1} \sqrt{[\phi_1(n) + \phi_2(y)]\,(1+y'^2)}\, dx$.

32. Find the form of an absolutely flexible nonextensible homogeneous rope of length l suspended at two points A and B.

33. Test for an extremum the functional $v(y(x)) = \int_0^{x_1} \frac{\sqrt{1+y'^2}}{\sqrt{y}}\, dx$, $y(0)=0$, $y(x_1) = y_1$.

Answer **any one** essay question out of 2 : **(1×10=10)**

34. Derive the equation of vibrations of a rectilinear bar.

35. Test for an extremum the functional

a) $v[y(x)] = \int_0^a \frac{y}{y'^2} dx$, $y(0) = 1$, $y(a) = b$.

$a > 0$, $0 < b < 1$.

b) Find the entremals of the functional

$v[y(x)] = \int (y'^2 + x^2)\, dx$, given that $\int_0^1 y^2 dx = 2$

$y(0) = 0$, $y(1) = 0$.

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2016
BHM 505 : Elective – I (D): CALCULUS OF VARIATIONS

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Define a linear functional.
2. State whether true or false "$y = x + c$ forms a field inside $x^2 + y^2 \leq 1$".
3. What you mean by Jacobi's condition.
4. For the functional $v = \int_{n_0}^{x_1} F(x, y, y')\, dx$, write the Euler equation.
5. Find the Ortogradsky equation for the functional $v(z) = \iint_D \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 dxdy$.
6. State true or false "Jacobi condition is necessary for achieving an extremum".
7. If $y = y_0(x)$, an extremum is attained and $y(x, \alpha)$ is a family of admissible curves, then $\frac{\partial}{\partial \alpha} v(n, \alpha)$ is ________
8. Write Jacobi's equation of second order.
9. Define central field.
10. "$F_y - \frac{d}{dx} F_{y'} = 0$ is sufficient for minimum of the elementary functional $v(g(x)) = \int_{x_0}^{x_1} F(x, y, y')\, dx$ $y(x_0) = y_0$, $y(x_1) = y_1$" – state whether the statement is true or false.

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. On what curves can the functional $v(y(x)) = \int_0^{\pi/2} (y')^3 - y^2 dx,\ y(0) = 0,\ y(\pi/2) = 1$ be extremised.

12. Does there exist an extremal for $v(y(x)) = \int_0^1 (y^2 + xy^1)\, dx,\ y(0) = 0,\ y(1) = 2.$

13. Find the extremals of $v(y(x)) = \int_{x_0}^{x1} y'(1 + x^2 y')\, dx$.

14. Check the validity of the problem $v(y(x)) = \int_{x_0}^{x_1} y dx + x dy,\ y(x_1) = y_1$.

15. Find the traversality condition for the functional $v(y(x)) = \int_{x_0}^{x_1} A(x, y)\sqrt{1 + y'^2}\, dx$, $A(x, y) \neq 0$.

16. Find the broken-line extremals (if they exist) of the functional $v = \int_0^a (y'^2 - y^2) dx,\ a > 0$.

17. Find the transversality condition for the extremal $v = \int_{x_0}^{x_1} A(x, y, z)\sqrt{1 + y'^2 + z'^2}\, dx$ if $z = g(x_1, y_1)$.

18. Are there any solutions with corner points in the extremum problem of the functional $v(y(x)) = \int_0^{x_2} (y'^4 - 6y'^2)\, dx,\ y(0) = 0,\ y(x_1) = y_1$.

19. Is the Jacobi condition fulfilled for extremal of the functional $V[y(x)] = \int_0^a (y'^2 + y^2 + x^3)\, dx$ that passes through the points (0, 0) and (a, 0).

20. Write the sufficient conditions for a functional v to acheive a weak extremum on a curve C.

21. Write a differential equation of the extremals of the functional $v(y(x)) = \int_0^{x_1} (p(x)y'^2 + q(x)y^2)\, dx$ given that $\int_0^{x_2} r(x)y^2 dx = 1,\ y(0) = 0,\ y(x_1) = 0.$

22. Formulate isoperimetric problem if the parametric represenation of a curve is given.

23. Give an example of a variational problem involving a conditional extremum.

24. Test for an extremum of the functional $v(y(x)) = \int_0^a y'^3 dx,\ y(0) = 0,$ $y(a) = b,\ a, b > 0.$

Answer **any 6** short essay questions. **(6x5=30)**

25. State and prove fundamental Lemma of calculus of variation.

26. Find a curve with specified boundary points whose rotation about the X-axis generates a surface of minimum area.

27. Derive the differential equation of free vibrations of a string.

28. Test for an extremum the functional $\int_0^{x_1} \frac{\sqrt{1 + y'^2}}{y} dx$ given that $y(0) = 0$ and $y_1 = x_1 - 5$.

29. Test for the extremum the functional $\int_0^{x_1} (y'^2 + z'^2 + 2yz)\, dx$ given that $y(0) = 0$, $z(0) = 0$ and the point (x_1, y_1, z_1) can move over the plane $x = x_1$.

30. Test for an extremum the functional $\int_0^a (6y'^2 - y'^4 + yy')\, dx,\ y(0) = 0,\ y(a) = b,$ $a, b > 0.$