31. Using Ostrogradsky-Hamilton principle, find the equations of motion of a system of particles of mass m_i (i = 1, 2, ...n) with coordinates (xi, yi, zi) acted upon by forces having force function U, given constraints

$\phi_j(t, x_1 \ldots x_n, y_1 \ldots y_n, z_1 \ldots z_n) = 0$, j = 1, 2, ... m.

32. Find the extremals of the isoperimetric problem $v[y(x)] = \int_{x_0}^{x_1} y'^2 \, dx$ given that $\int_{x_0}^{x_1} y dx = a$.

33. Test for an extremum the functional $\int_0^a \left(6y'^2 - y'^4 + yy'\right) dx$, y(0) = 0, y (a) = b. a, b > 0.

Answer **any one** essay question out of **2**. **(1×10=10)**

34. Derive the transversality condition in the problem of finding the functional

$$v = \int_{x_0}^{x_1} F(x, y, z, y', z') dx$$., for an extremum.

35. a) Derive the equation of vibrations of a rectangular bar.

b) Test the extremum of the functional $S = \int_0^{x_1} \frac{\sqrt{1+y^2}}{\sqrt{y}} dx$, y(0) = 0, $y(x_1) = y_1$.

K15U 0349

Reg. No. : ....

Name : ....

V Semester B.Sc. (Hon's) (Mathematics) Degree (Regular) Examination, November 2015

BHM 505 : CALCULUS OF VARIATION (Elective – I (D))

Time : 3 Hours Max. Marks : 80

Answer **all the ten** questions. **(10×1=10)**

1. Write the Euler's equation of the functional $v(y(x)) = \int_0^1 (y'^2 + 12xy) dx$.
2. If for every continuous function f(x), $\int_{x_0}^{x_1} f(x) g(x) = 0$ and g(x) is continuous on $[x_0, x_1]$, then g(x) is ____________
3. Give an example of a functional which depend on functions of several variable.
4. State true or false "The pencil of sinusoids y = csinx $0 \le x \le a$, $a < \pi$ forms a central field".
5. Write Jacobi's equation of second order.
6. Write one of the sufficient conditions for a functional to achieve a weak extremum.
7. Write Ostrogradsky-Hamilton principle.
8. Find the Ostrogradsky equation for the functional $v(z) = \iint_D \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 dx\,dy$.
9. State true or false "Jacobi condition is not necessary for achieving an extremum".
10. Find the Euler equation for the functional $v = \int_{x_0}^{x_1} F(x, y, y') dx$.

P.T.O.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Find the extremals of the functional

$$v(y(x), z(x)) = \int_0^{\pi_2} (y'^2 + z'^2 + 2yz)\,dx .$$

$y(0)=0,\ y(\pi_2)=1,\ z(0)=0,\ z(\pi_2)=-1$

12. Find if any, an extremal Satisfying

$$v[y(x)] = \int_0^1 \left(y^2 + x^2 y^1 \right) dx ,\ y(0) = 0,\ y(1) = 2.$$

13. State Fundamental Lemma of Calculus of variations.

14. Find the traversality condition for the functional

$$v(y(x)) = \int_{x_0}^{x_1} A(x,y)\sqrt{1+y'^2}\,dx \quad A(x,y) \neq 0 .$$

15. Find the broken line extremals of the functional $v = \int_{x_0}^{x_1} y'^2(1-y')^2\,dx$.

16. Test for an extremum the functional $v[y(x)] = \int_0^1 \left(y'^2 - y^2\right)dx,\ y(0)=0, y(1)=0.$

17. Find a curve of given length *l* bounding, together with a given curve $y = f(x)$ the maximum area cross-hatched.

18. Write the sufficient conditions for a functional v to achieve a strong extremum on a curve C.

19. Is the Jacobi condition fulfilled for the extremal of the functional

$$v = \int_0^1 \left(y'^2 + y^2 + x^2 \right) dx$$ that passes through (0, 0) and (1, 0).

20. Find the extremal distance between two surfaces $z = \phi(x, y)$ and $z = \psi(x, y)$.

21. Find the Ostrogradsky equation of $v(z(x,y)) = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zf(x,y) \right] dx\,dy$.

22. Find the extremal of the functional $v[y(x)] = \int_0^1 \left(1 + y''^2\right) dx,\ y(0)=0,\ y'(0)=1$

$y(1) = 1,\ y'(1) = 1.$

23. Is the variational problem $v(y(x)) = \int_{x_0}^{x_1} (y + xy')\,dx ,\ y(x_0) = y_0,\ y(x_1) = y_1$ meaningful.

24. Formulate isoperimetric problem if the parametric representation of the curve is given.

Answer **any 6** short essay questions out of **9**. **(6×5=30)**

25. Find the extremals of the functional $v[y(x)] = \int_0^{x_1} \frac{\sqrt{1+y'^2}}{y}\,dx$., $y(0) = 0,\ y_1 = x_1 - 5.$

26. Derive the differential equation of free vibrations of a string.

27. Derive a method to find extremal of the functional

$$v[y(x)] = \int_{x_0}^{x_1} \left(M(x,y) + N(x,y)\frac{dy}{dx} \right) dx .$$

28. Find the extremals of the functional $v(y(x), z(x)) = \int_{x_0}^{x_1} F(y', z')\,dx$

29. Find the broken line extremals of the functional $v = \int_{x_0}^{x_1} y'^2 (1-y')^2\,dx$

30. Test the functional for extrema : $v(y(x)) = \int_{-1}^{2} y'(1 + x^2 y')\,dx ,\ y(-1) = 1,\ y(2) = 4.$