Reg. No. : ....

Name : ....

K16U 2586

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2016
BHM501 : ALGEBRA, ALGORITHMS & DATA STRUCTURES

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Find f (x) + g (x) and f (x). g (x) in $\mathbb{Z}_2$ [x] if f (x) = x + 1 and g (x) = x + 1.
2. What are the units in $\mathbb{Z}_7$[x] ?
3. State true or false : a polynomial f (x) of degree n with coefficients in a field F can have atmost n zeros in F.
4. Give an ideal for the ring $\mathbb{Z}$, the set of all integers.
5. What do you mean by a maximal ideal ?
6. What do you mean by a queue ?
7. Write any three data structure operations.
8. What is the complexity of a binary search algorithm ?
9. Suppose multy dimensional arrays A and B are declared using A(– 2 : 2, 2 : 22) and B(1 : 8, – 5 : 5, –10 : 5). Find the length of each dimension.
10. Give examples for triangular and tridiagonal matrices.

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. Prove that if D is an integral domain, then D[x] is an integral domain.
12. Factorize $x^4 + 3x^3 + 2x + 4$ in $\mathbb{Z}_5$[x].
13. Show that $x^3 + 3x + 2$ is irreducible over $\mathbb{Z}_5$.

P.T.O.

14. Let F be the ring of all functions mapping R into R, and let N be the subring of all functions f such that f (2) = 0. Is N an ideal in F ? Justify your answer.

15. Let R be a ring with unity 1. Prove that the map $\phi: \mathbb{Z} \rightarrow R$ given by $\phi(n) = n.1$ for $n \in \mathbb{Z}$ is a homomorphism of $\mathbb{Z}$ into R.

16. Show that the real number $\alpha = \sqrt{1+\sqrt{3}}$ is algebraic over $\mathbb{Q}$. Also find deg $(\alpha, \mathbb{Q})$.

17. Find the degree and a basis for $\mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{18}\right)$ over $\mathbb{Q}$.

18. What do you mean by worst case complexity of an algorithm ? What is the worst case complexity of linear search algorithm ?

19. Write bubble sort algorithm.

20. Write a note on big 'O' notation.

21. Write a note on subalgorithms.

22. What do you mean by linear arrays ? Write an algorithm for deleting an item from a linear array.

23. Write binary search algorithm to find the location of an ITEM in a sorted ARRAY.

24. Explain the representation of two dimensional arrays in memory.

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. Let F be a field of quotients of D and let L any field containing D. Then prove that there exists a map $\psi: F \rightarrow L$ that gives an isomorphism of F with a subfield of L such that $\psi(a) = a$ for $a \in D$.

26. State and prove Eisenstein Criterion.

27. If F is a field, then every nonconstant polynomial $f(x) \in F[x]$ can be factored in F[x] into a product of irreducible polynomials, the irreducible polynomials being unique except for order and for unit factors in F.

28. If F is a field, prove that every ideal in F[x] is principal.

29. Prove that the field C of complex numbers is an algebraically closed field.

30. Briefly explain the idea of complexity of an algorithm.

31. Write an algorithm to print the prime numbers less than N.

32. Consider the DATA :
11, 22, 30, 33, 40, 44, 55, 60, 66, 77, 80, 88, 99.
Apply binary search to DATA for searching the items 40 and 85.

33. Suppose A is a sorted array with 200 elements and suppose a given element x appears with the same probability in any place in A. Find the worst-case running time f(n) and the average-case running time g(n) to find x in A using the binary search algorithm.

Answer **any one** essay questions out of 2. **(1×10=10)**

34. Let F be a field and let f(x) be a nonconstant polynomial in f[x]. Prove that there exists an extension field E of F and an $\alpha \in E$ such that $f(\alpha) = 0$.

35. a) Explain control structures with examples.

    b) Describe briefly the difference between local variables, parameters and global variables.

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