Reg. No. : ....

Name : ....

V Semester B.Sc. (Hon's) (Mathematics) Degree (Regular)
Examination, November 2015
BHM 501 : ALGEBRA, ALGORITHMS AND DATA STRUCTURES

Time : 3 Hours

Max. Marks : 80

Answer **all** the **ten** questions.

(10×1=10)

1. Find $f(x) + g(x)$ and $f(x) \cdot g(x)$ in Z_8 [x] if $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$.

2. What do you mean by an irreducible polynomial in $F[x]$?

3. State true or false : If F is a filed, the units of F [x] are precisely the nonzero elements of F.

4. What do you mean by projection homomorphism in rings ?

5. Give an example for a prime ideal in $\mathbb{Z}$, the set of all integers.

6. What do you mean by a tree ?

7. What do you mean by data structure ?

8. What is the complexity of a linear search algorithm ?

9. Suppose multi dimensional arrays A and B are declared using A (–2 : 2, 2 : 22) and B (1 : 8, –5 : 5, –10 : 5). Find the number of elements in A and B.

10. What do you mean by sparse matrices ?

Answer **any 10** short answer questions out of **14**.

(10×3=30)

11. Prove that $x^2 - 2$ has no zeros in the rational numbers.

12. Apply division algorithm for $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$ and $g(x) = x^2 + 2x - 3$

in $\mathbb{Z}_7[x]$.

P.T.O.

13. Show that $25x^5 - 9x^4 - 3x^2 - 12$ is irreducible over $\mathbb{Q}$.

14. Prove that a ring homomorphism $\phi : R \to R'$ is a one-to-one map if and only if Ker $(\phi) = \{0\}$.

15. Let R be a ring with unity and N be an ideal of R containing a unit. Prove that N = R.

16. Show that the real number $\alpha = \sqrt{1 + \sqrt{3}}$ is algebraic over $\mathbb{Q}$. Also find deg $(\alpha, \mathbb{Q})$.

17. Show that $\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$ is a basis for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.

18. Explain the term space-time tradeoff of algorithms.

19. What do you mean by complexity of an algorithm ?

20. Write a note on big 'O' notation.

21. Explain different categories of subalgorithms.

22. What do you mean by linear arrays ? Write an algorithm for traversing linear arrays.

23. Write a note on binary search algorithm in linear arrays. What are the limitations of binary search algorithms ?

24. Write an algorithm for matrix multiplication.

Answer **any 6** short answer questions out of **9**. (6×5=30)

25. Prove that the set R[x] of all polynomials in an indeterminate x with coefficients in a ring R is a ring under polynomial addition and multiplication. If R is commutative then prove that R[x] is also commutative.

26. State and prove division algorithm in F[x].

27. Prove that the polynomial

$$\Phi_p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \dots + x + 1 \text{ is irreducible over } \mathbb{Q} \text{ for any prime p.}$$

28. State and prove fundamental homomorphism theorem for rings.

29. An ideal $\langle p(x) \rangle \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible over F.

30. Draw a tree diagram for the algebraic expression $(7x + y)(5a - b)^3$.

31. Write an algorithm to find the root of a quadratic equation.

32. Suppose the following numbers are stored in an array A :

    32, 51, 27, 85, 66, 23, 13, 57

    Apply bubble sort to sort these numbers in ascending order.

33. Suppose A is a sorted array with 200 elements and suppose a given element x appears with the same probability in any place in A. Find the worst-case running time f(n) and the average-case running time g(n) to find x in A using the binary search algorithm.

Answer **any one** essay questions out of **2**. (1×10=10)

34. Prove that every integral domain D can be enlarged to a filed F such that every element of F can be expressed as a quotient of two elements of D.

35. What do you mean by an algorithm ? Give an example. What are the different parts of an algorithm ?