K21U 0223

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2020
(2016 Admission Onwards)
BHM 503 : ADVANCED DISCRETE MATHEMATICS

Time : 3 Hours　　　　Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark :

1. State the Hall's condition.
2. State the marriage theorem.
3. State the 1-factorization conjecture.
4. Define the generating function for a given sequence of real numbers $a_0, a_1, a_2, \ldots$
5. Define the exponential generating function for a given sequence of real numbers $a_0, a_1, a_2, \ldots$ **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks :

6. Define the adjacency matrix A and the out degree matrix B of a digraph. What can you say about the matrix B – A ?
7. When do you say a component of a graph to be even ? Give an example.
8. Explain the concept of edge independence number with a suitable example.
9. Explain the concept of Hamiltonian factorable graph with a suitable example.
10. State the principle of Inclusion and Exclusion.

P.T.O.

11. Define the number of derangements of a set with n elements and calculate the number of derangements of 1, 2, 3, 4.

12. Determine the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + ...)^4$.

13. If c_k represents the number of ways to make change for k rupees, using Re. 1, Rs. 2, Rs. 5, Rs. 10 and Rs. 100, find the generating function for c_k.

14. Determine the sequence generated by $(1 - 4x)^{-1/2}$. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks :

15. Explain the Konigsberg bridge problem.

16. Construct the de Bruijn digraph B(3, 2). Find an Eulerian circuit in the above graph and write down the corresponding de Bruijn sequence.

17. Define the terms Hamiltonian path and Hamiltonian cycle. Also give an example of a graph which contains a Hamiltonian path but not a Hamiltonian cycle.

18. Prove that every bridgeless cubic graph contains a perfect matching.

19. Explain the concepts of maximum matching, maximal matching and perfect matching with suitable examples.

20. Prove that, for each positive integer k, the complete graph K_{2k} is 1-factorable. Illustrate the above result with a 1-factorization of K_6.

21. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2 or 3 or 5.

22. Find the number of permutations of a, b, c, ..., x, y, z in which none of the patterns byte, car, dog or pun occurs.

23. At a 12 week conference in Discrete Mathematics, Sherly met seven of her friends from her college. During the conference she met each friend at lunch 35 times, every pair of them 16 times, every trio eight times, every foursome four times, each set of five twice and each set of six once, but never all seven at once. If she had lunch every day during the 84 days of the conference, did she ever have lunch alone ?

24. Find the number of integer solutions for the following equations :
    i) $c_1 + c_2 + c_3 + c_4 = 20$ if $0 \leq c_i$ for all $1 \leq i \leq 4$ with c_2 and c_3 even.
    ii) $c_1 + c_2 + c_3 + c_4 + c_5 = 30$ if $2 \leq c_1 \leq 4$ and $3 \leq c_i \leq 8$ for all $2 \leq i \leq 5$.

25. In how many ways can a police captain distribute 26 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight ?

26. A ship carries 48 flags, 12 each of the colours red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of the signals have at least three white flags or no white flags at all ? **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks :

27. Prove that the Petersen graph is not Hamiltonian.

28. Explain the concept of a system of distinct representatives. State and prove a necessary and sufficient condition for a collection $\{S_1, S_2, ..., S_n\}$ of finite nonempty sets to be a system of distinct representatives.

29. State the principle of Inclusion and Exclusion, using Mathematical Induction or otherwise prove it.

30. Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ subject to $1 \leq x_1 \leq 5, -3 \leq x_2 \leq 4, 0 \leq x_3 \leq 5, 5 \leq x_4 \leq 9$. **(2×6=12)**
