29. Consider a set S, with $|S| = N$, and conditions $c_i, 1 \le i \le t$, each of which may be satisfied by some of the elements of S. Then prove that, for each $1 \le m \le t$, the number of elements in S that satisfy exactly m of the conditions $c_1, c_2, ..., c_t$ is given by.

$$Em = Sm - \binom{m+1}{1} S_{m+1} + \binom{m+2}{2} S_{m+2} - ... + (-1)^{t-m} \binom{t}{t-m} S_t.$$

Also deduce the principle of Inclusion and Exclusion.

30. Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ subject to $1 \le x_1 \le 5, -2 \le x_2 \le 4, 0 \le x_3 \le 5, 3 \le x_4 \le 9$.

Reg. No. : .....

Name : .....

V Semester B.Sc. Hon's (Mathematics) Degree(Reg./Supple./Improv.)

Examination, November-2019

(2016 Admission Onwards)

BHM 503 : ADVANCED DISCRETE MATHEMATICS

Time : 3 hours Max. Marks : 60

SECTION - A

Answer any **4** questions out of **5** questions. Each question carries **1** Mark. **(4×1=4)**

1. Define a matching.
2. State the Hall's condition.
3. State Ringel-Kotzig's conjecture.
4. Define the generating function for a given sequence of real numbers $a_0, a_1, a_2, ...$
5. Write down the exponential generating function for a given sequence of real numbers.1,1,1,3,1,1,1,,,.....

SECTION - B

Answer any **6** questions out of **9** questions. Each question carries **2** Marks. **(6×2=12)**

6. Define the outdegree matrix of a digraph and give an example.
7. When do you say a component of a graph to be odd ? Give an example.
8. Explain the concept of maximal matching with a suitable example.
9. Explain the concept of factorization with a suitable example.

10. Define the number of derangements of a set with n elements and calculate the number of derangements of 1,2,3,4,5.

11. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2 or 3.

12. Determine the coefficient of x^{17} in $f(x) = (x^2 + x^3 + x^4 + ...)^4$

13. If c_k represents the number of ways to make change for k rupees, using Rs 1, Rs 2, Rs 5 and Rs 10, find the generating function for c_k.

14. How many integer solutions are there for the equation $c_1 + c_2 + c_3 + c_4 = 20$ if $0 \leq c_i$ for all $1 \leq i \leq 4$.

SECTION - C

Answer any **8** questions out of **12** questions . Each question carries **4** Marks.
(8×4=32)

15. Define the terms Eulerian circuit and Eulerian trial. Also give an example of a graph that contains neither an Eulerian trial nor an Eulerian circuit .

16. Construct the de Bruijn digraph B(2,4). Find an Eulerian circuit in the above graph and write down the corresponding de Bruijn sequence.

17. Define the terms Hamiltonian path and Hamiltonian cycle. Also give an example of a graph which contains a Hamiltonian path and a Hamiltonian cycle.

18. Explain the concepts edge independence number and lower edge independence number with suitable examples.

19. For every nonempty graph G, prove that $\alpha_0'(G) \leq \alpha'(G) \leq 2\alpha_0'(G)$.

20. Illustrate a cyclic k_3-decomposition of k_7.

21. Let *A*= {1,2,3,4} and $B = \{a,b,c,d,e,f\}$. Determine the number of one-to-one functions $f : A \to B$ where none of the following conditions are satisfied: $c_1 : f(1) = a$ or b $c_2 : f(2) = c$ $c_3 : f(3) = c$ or d $c_4 : f(4) = d,e$ or f .

22. Determine the number of positive integers n where $1 \leq n \leq 300$ and n is not divisible by 5 or 7 or 11.

23. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun, or byte occurs ?

24. Find the number of integer solutions for the following equations:

    (i) $c_1 + c_2 + c_3 + c_4 = 20$ if $0 \leq c_i$ for all $1 \leq i \leq 4$ with c_2 and c_3 even.

    (ii) $c_1 + c_2 + c_3 + c_4 + c_5 = 30$ if and $2 \leq c_1 \leq 4$ and $3 \leq c_i \leq 8$ for all $2 \leq i \leq 5$.

25. In how many ways can a police captain distribute 22 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight?

26. A ship carries 48 flags, 12 each of the colours red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of the signals have at least three white flags or no white flags at all ?

SECTION - D

Answer any **2** questions out of **4** questions . Each question carries **6** Marks.
(2×6=12)

27. Let G be a graph of order $n \geq 3$. if deg u + deg $v \geq n$ for each pair u,v of nonadjacent vertices of G, then prove that G is Hamiltonian.

28. Let G be a bipartite graph with partite sets U and *W*, where $|U| \leq |W|$. Then prove that U can be matched to a subset of W if and only if Hall's condition is satisfied.