K19U 3027



29. Consider a set S, with |S| = N, and conditions $c_i, 1 \le i \le t$, each of which may be satisfied by some of the elements of S. Then prove that, for each $1 \le m \le t$, the number of elements in S that satisfy exactly m of the conditions $c_1, c_2,, c_t$ is given by.

(4)

$$Em = Sm - \binom{m+1}{1}S_{m+1} + \binom{m+2}{2}S_{m+2} - \ldots + (-1)^{t-m}\binom{t}{t-m}S_t.$$
 Also deduce the principle of Inclusion and Exclusion.

30. Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ subject to $1 \le x_1 \le 5, -2 \le x_2 \le 4, 0 \le x_3 \le 5, 3 \le x_4 \le 9$.



K19U 3027

Reg. No. :

V Semester B.Sc. Hon's (Mathematics) Degree(Reg./Supple./Improv.)

Examination, November-2019

(2016 Admission Onwards)

BHM 503: ADVANCED DISCRETE MATHEMATICS

Time: 3 hours

Max. Marks: 60

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 Mark.

(4×1=4)

- Define a matching.
- 2. State the Hall's condition.
- 3. State Ringel-Kotzig's conjecture.
- 4. Define the generating function for a given sequence of real numbers a_0, a_1, a_2, \dots
- Write down the exponential generating function for a given sequence of real numbers.1,1,1,3,1,1,1,......

SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 Marks.

(6×2=12)

- Define the outdegree matrix of a digraph and give an example.
- 7. When do you say a component of a graph to be odd? Give an example.
- 8. Explain the concept of maximal matching with a suitable example.
- 9. Explain the concept of factorization with a suitable example.

P.T.O.



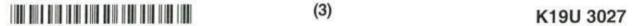


- **10.** Define the number of derangements of a set with *n* elements and calculate the number of derangements of 1,2,3,4,5.
- 11. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2 or 3.
- 12. Determine the coefficient of x^{17} in $f(x) = (x^2 + x^3 + x^4 + ...)^4$
- 13. If c_k represents the number of ways to make change for k rupees, using Rs 1, Rs 2, Rs 5 and Rs 10, find the generating function for c_k .
- **14.** How many integer solutions are there for the equation $c_1 + c_2 + c_3 + c_4 = 20$ if $0 \le c_i$ for all 1 < i < 4.

SECTION - C

Answer any 8 questions out of 12 questions. Each question carries 4 Marks. (8×4=32)

- 15. Define the terms Eulerian circuit and Eulerian trial. Also give an example of a graph that contains neither an Eulerian trial nor an Eulerian circuit.
- 16. Construct the de Bruijn digraph B(2,4). Find an Eulerian circuit in the above graph and write down the corresponding de Bruijn sequence.
- Define the terms Hamiltonian path and Hamiltonian cycle. Also give an example of a graph which contains a Hamiltonian path and a Hamiltonian cycle.
- 18. Explain the concepts edge independence number and lower edge independence number with suitable examples.
- **19.** For every nonempty graph G, prove that $\alpha'_0(G) \le \alpha'(G) \le 2\alpha'_0(G)$.
- **20.** Illustrate a cyclic k_3 -decomposition of k_7 .



- **21.** Let $A = \{1,2,3,4\}$ and $B = \{a,b,c,d,e,f\}$. Determine the number of one-to-one functions $f : A \to B$ where none of the following conditions are satisfied: $c_1 : f(1) = a$ or $b : c_2 : f(2) = c : c_3 : f(3) = c$ or $d : c_4 : f(4) = d,e$ or $f : c_5 : f(4) = d,e$ or f : f(4) : f(4) = d,e or f : f(4) :
- **22.** Determine the number of positive integers n where $1 \le n \le 300$ and n is not divisible by 5 or 7 or 11.
- 23. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun, or byte occurs?
- 24. Find the number of integer solutions for the following equations:
 - (i) $c_1 + c_2 + c_3 + c_4 = 20$ if $0 \le c_i$ for all $1 \le i \le 4$ with c_2 and c_3 even.
 - (ii) $c_1 + c_2 + c_3 + c_4 + c_5 = 30$ if and $2 \le c_1 \le 4$ and $3 \le c_i \le 8$ for all $2 \le i \le 5$.
- 25. In how many ways can a police captain distribute 22 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight?
- 26. A ship carries 48 flags, 12 each of the colours red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of the signals have at least three white flags or no white flags at all?

SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 Marks. (2×6=12)

- **27.** Let G be a graph of order $n \ge 3$. if deg $u + \deg v \ge n$ for each pair u, v of nonadjacent vertices of G, then prove that G is Hamiltonian.
- **28.** Let G be a bipartite graph with partite sets U and W, where $|U| \le |W|$. Then prove that U can be matched to a subset of W if and only if Hall's condition is satisfied.

P.T.O.