K21U 0222

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2020
(2016 Admission Onwards)
BHM 502 : ADVANCED COMPLEX ANALYSIS

Time : 3 Hours　　　　Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Find $\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$.
2. Define simply connected region and give an example.
3. If $\sin(1 + z) = \sum_{n=0}^{\infty} a_n z^n$, find a_3.
4. Write a singular point of the principal branch of Logz.
5. State argument principle.　　　　**(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Let C denote a contour of length L and suppose f(z) is piecewise continuous on C and $|f(z)| \leq M$ for all z in C at which f(z) is defined. Show that

$$\left|\int_C f(z)dz\right| \leq ML.$$

7. Evaluate $\int_C ze^{-z}\,dz$ where C is the positively oriented circle $|z| = 1$.
8. If a functions is analytic at a given point, then prove that its derivatives of all orders are analytic there too.

P.T.O.

9. Write the Maclaurin series of $z^2 \sinh z$.

10. Find $\lim_{n \to \infty} z_n$, where $z_n = 1 + i\frac{(-1)^n}{n^2}$, n = 1, 2, . . .

11. Identify the singularity of $f(z) = \frac{1-\cos z}{z^2}$ at z= 0.

12. If z_0 is a pole of f then prove that $\lim_{z \to z_0} f(z) = \infty$.

13. Let C denote the positively oriented circle |z| = 1.

Find Δ_c avg f(z) where $f(z) = z^2$.

14. State Jordan's Lemma. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Evaluate $\int_C \bar{z}\, dz$ where $C = \{z : z = ze^{i\theta}, -\pi/2 \le \theta \le \pi/2\}$ with positive orientation.

16. State and prove fundamental theorem of algebra.

17. Suppose $f(z) = (z + 1)^2$ and R is the closed triangular region at the points z= 0, z = 2 and z = i. Find the maximum value of |f(z)| in R.

18. Prove or disprove : "a sequence of complex numbers $z_n = r_n e^{i\theta_n}$ converges to $z = re^{i\theta}$ if and only if r_n converges to r and θ_n converges to θ.

19. Represent the function $f(z) = \frac{z+1}{z-1}$ by its Laurant series in the domain $| < |z| < \infty$.

20. Show that $f(z) = \begin{cases} \frac{e^z - 1}{z}, & \text{when } z \neq 0 \\ 1 & \text{when } z = 0 \end{cases}$ is an entire function.

21. State and prove residue theorem.

22. Suppose $f(z) = \frac{\phi(z)}{(z-z_0)^n}$ where φ(z) is analytic and $\phi(z_0) \neq 0$. Find $\operatorname{Res}_{z=z_0} f(z)$ for m ≥2.

23. If z_0 is a removable singular point of a function f then prove that f is analytic and bounded in some deleted neighbourhood $0 < |z - z_0| < \epsilon$ of z_0.

24. Using residues, evaluate $\int_0^\infty \frac{x^2}{x^0 + 1} dx$.

25. Prove that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.

26. Determine the number of roots of the equation

$2z^5 - 6z^2 + z + 1 = 0$ in $1 \le |z| < 2$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Evaluate $\int_C z^{1/2} dz$ where C is any contour from z = – 3 to z = 3 that, except for its end points lie above the x-axis.

28. If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1 \neq z_0$ then prove that it is absolutely convergent at each point z in the open disk $|z - z_0| < R$ where $R = |z_1 - z_0|$.

29. Suppose that a function f is analytic throughout the finite plane except for a finite number of singular points $z_1, \ldots, z_n$. Show that $\sum_{i=1}^{n} \operatorname{Res}_{z=z_i} f(z) + \operatorname{Res}_{z=\infty} f(z) = 0$.

30. Evaluate $\int_{-\infty}^{\infty} \frac{\cos x\, dx}{(x^2 + a^2)(x^2 + b^2)}$. **(2×6=12)**