Explosive for the state of the state of an amount of the provents of the analyses and bounded to pome delated neighbourhood of $c | x - x_0 | c = c | x_0$.

24. Using residues, available $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1} dx$.

25. Prove that $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2} dx$.

26. Contends on auchieu of roots of the equation.

27. Substitute of the substitute of the state of the equation.

28.ECTION - D.

27. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2} dx$ where $\int_{-\infty}^{\infty} \frac{dx}{x^2} dx$ is proved the x-axis.

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V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.)
Examination, November 2020

(2016 Admission Onwards)
BHM 502 : ADVANCED COMPLEX ANALYSIS

Time: 3 Hours

Max. Marks: 60

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark.

1. Find
$$\int_{1}^{2} \left(\frac{1}{t} - i\right)^{2} dt$$

2. Define simply connected region and give an example.

3. If
$$\sin (1 + z) = \sum_{n=0}^{\infty} a_n z^n$$
, find a_3 .

4. Write a singular point of the principal branch of Logz.

5. State argument principle.

(4×1=4)

SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 marks.

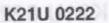
 Let C denote a contour of length L and suppose f(z) is piecewise continuous on C and |f(z)| ≤ M for all z in C at which f(z) is defined. Show that

$$\left|\int\limits_{C}f(z)dz\right|\leq ML.$$

7. Evaluate $\int_{C} ze^{-z} dz$ where C is the positively oriented circle |z| = 1.

8. If a functions is analytic at a given point, then prove that its derivatives of all orders are analytic there too.

P.T.O.





- 9. Write the Maclaurin series of z²sinhz.
- 10. Find $\lim_{n\to\infty} z_n$, where $z_n = 1 + i \frac{\left(-1\right)^n}{n^2}$, $n = 1, 2, \ldots$
- 11. Identify the singularity of $f(z) = \frac{1 \cos z}{z^2}$ at z = 0.
- 12. If z_0 is a pole of f then prove that $\lim_{z \to z_0} f(z) = \infty$.
- 13. Let C denote the positively oriented circle |z| = 1. Find Δ avg f(z) where f(z) = z^2 .

14. State Jordan's Lemma. (6x2=12)

SECTION - C

Answer any 8 questions out of 12 questions. Each question carries 4 marks.

- 15. Evaluate $\int_{0}^{\infty} z \, dz$ where $C = \{z : z = ze^{i\theta}, -\pi/2 \le \theta \le \pi/2\}$ with positive orientation.
- 16. State and prove fundamental theorem of algebra.
- 17. Suppose $f(z) = (z + 1)^2$ and R is the closed triangular region at the points z = 0, z = 2 and z = i. Find the maximum value of |f(z)| in R.
- 18. Prove or disprove : "a sequence of complex numbers $z_n = r_n e^{i\theta_n}$ converges to $z = re^{i\theta}$ if and only if r_n converges to r and θ_n converges to θ .
- 19. Represent the function $f(z) = \frac{z+1}{z-1}$ by its Laurant series in the domain $|z| < \infty$.
- 20. Show that $f(z) = \begin{cases} \frac{e^z 1}{z}, & \text{when } z \neq 0 \\ 1, & \text{when } z = 0 \end{cases}$ is an entire function.
- 21. State and prove residue theorem.
- 22. Suppose $f(z) = \frac{\phi(z)}{(z-z_0)^n}$ where $\phi(z)$ is analytic and $\phi(z_0) \neq 0$. Find Res f(z) for $m \ge 2$.

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23. If z₀ is a removable singular point of a function f then prove that f is analytic and bounded in some deleted neighbourhood $0 < |z - z_0| < \epsilon$ of z_0 .

24. Using residues, evaluate
$$\int_{0}^{\infty} \frac{x^2}{x^0 + 1} dx$$
.

25. Prove that
$$\int_{0}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}.$$

26. Determine the number of roots of the equation

$$2z^5 - 6z^2 + z + 1 = 0$$
 in $1 \le |z| < 2$. (8×4=32)

SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 marks.

27. Evaluate $\int z^{\frac{1}{2}} dz$ where C is any contour from z = -3 to z = 3 that, except for its end points lie above the x-axis.

28. If a power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges when $z=z_1 \neq z_0$ then prove that it is absolutely convergent at each point z in the open disk $|z-z_0|$ <R where $R = |z_1 - z_0|$.

29. Suppose that a function f is analytic throughout the finite plane except for a finite number of singular points z_1, \ldots, z_n . Show that $\sum_{z=z}^n \operatorname{Res}_{z=z} f(z) + \operatorname{Res}_{z=\infty} f(z) = 0$.

30. Evaluate
$$\int_{-\infty}^{\infty} \frac{\cos x \, dx}{(x^2 + a^2)(x^2 + b^2)}$$
 (2x6=12)