SECTION - D

Answer any **2** questions out of **4** questions . Each question carries **6** Marks.

(2×6=12)

27. Prove that $\int_c \frac{e^{az}}{z}dz = 2\pi i$ where $a>0$ and C is the circle $\{z : z = e^{i\theta}, -\pi \le \theta \le \pi\}$.

Hence or otherwise prove that $\int_0^{\pi} e^{a\cos\theta}\cos(a\sin\theta)d\theta = \pi$.

28. State and prove Laurent's theorem.

29. Evaluate $\int_c \frac{\cosh \pi z}{z(z^2+1)}dz$ where C is the circle $|z|=2$ described in the positive sense.

30. Evaluate $\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta}, |a|<1$.

K19U 3026

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree(Reg./Supple./Improv.)

Examination, November-2019

(2016 Admission Onwards)

BHM 502 : ADVANCED COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION - A

Answer any **4** out of **5** questions . Each question carries **1** Mark. **(4×1=4)**

1. Evaluate $\int_0^{\pi/6} e^{i2t}\, dt$.

2. Write Cauchy's integral formula.

3. Write the Laurent series expansion of $e^{1/z}$.

4. Find the order of the pole at z = 0 of the function $f(z) = \frac{\sinh z}{z^4}$.

5. State Rouche's theorem.

SECTION - B

Answer any **6** questions out of **9** questions . Each question carries **2** Marks.

(6×2=12)

6. Suppose $c = \{z : z = 2e^{i\theta}, 0 \le \theta \le \pi/2\}$ with positive orientation. Show that $\left|\int_c \frac{z+4}{z^3-1}dz\right| \le \frac{6\pi}{7}$.

7. If a function f is analytic throughout a simply connected domain D, then prove that $\int_c f(z)dz = 0$ for any contour C lying in D.

8. Evaluate $\int_c ze^{-z}dz$ where C is the circle $|z| = 1$

9. Prove that absolute convergence of a series of complex numbers implies the convergence of that series.

10. Represent the function $f(z) = \frac{z+1}{z-1}$ by maclaurin series.

11. Find the order of the zero at z = 0 of $f(z) = z^2(\cos z - 1)$.

12. Let C denote the circle $|z| = 1$ with positive orientation. Find $\Delta_c \arg f(z)$ where

$f(z) = \frac{z^3+2}{z}$.

13. State Argument principle.

14. Find the residue of $f(z) = \frac{z+1}{(z-1)^3(z-2)}$ at z = 2.

SECTION - C

Answer any **8** questions out of **12** questions. Each question carries **4** Marks. **(8×4=32)**

15. Evaluate $\int_c z^{a-1}dz$ where C is the positively oriented circle $\{z : z = Re^{i\theta}, -\pi \le \theta \le \pi\}$

16. Suppose that a function f is analytic inside and on a positively oriented circle C_R centered at z_0 with radius R. if M_R in the maximum value of $|f(z)|$ on C_R, then prove that $\left|f^{(n)}_{(z_0)}\right| \le \frac{n!\, M_R}{R^n}$, $n = 1,2,3,\ldots$

17. Find $\int_c \frac{1}{(z^2+4)^2}dz$ where C is the positively oriented circle $|z-i| = 2$.

18. Show that if $|z-i| < \sqrt{2}$, $\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}$.

19. Find the Laurent series of $f(z) = \frac{1}{(z-1)(z-2)}$ in $1 < |z| < 2$.

20. If z_1 is a point inside the circle of convergence $|z - z_0| = R$ of a power series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$ then prove that the series must be uniformly convergent in the closed disk $|z-z_0| \le R_1$ where $R_1 = |z_0 - z_1|$.

21. If a function f is analytic every where in the infinite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then prove that $\int_c f(z)dz = 2\pi i \operatorname*{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right]$.

22. Suppose $f(z) = \frac{(\log z)^3}{z^2+1}$ where $\log z = \log r + i\theta,\ 0 < \theta < 2\pi, r > 0$. Find the residues at the poles.

23. Evaluate $\int_0^{\infty} \frac{dx}{x^2+1}$ using residues.

24. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$

25. Determine the number of zeros of $z^4 + 3z^3 + 6$. inside the circle $|z| = 2$.

26. State and prove Residue theorem.

P.T.O.