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SECTION - D

(4)

Answer any 2 questions out of 4 questions . Each question carries 6 Marks. (2×6=12)

- **27.** Prove that $\int_{c}^{e^{az}} dz = 2\pi i$ where a > 0 and C is the circle $\{z : z = e^{i\theta}, -\pi \le \theta \le \pi\}$. Hence or otherwise prove that $\int_{0}^{z} e^{a\cos\theta} \cos(a\sin\theta) d\theta = \pi$.
- 28. State and prove Laurent's theorem.
- **29.** Evaluate $\int_{c}^{\infty h\pi z} \frac{dz}{z(z^2+1)} dz$ where C is the circle |z|=2 described in the positive sense.
- 30. Evaluate $\int_{0}^{2\pi} \frac{d\theta}{1 + a\cos\theta}, |a| < 1.$



K19U 3026

Name :

Reg. No.:....

V Semester B.Sc. Hon's (Mathematics) Degree(Reg./Supple./Improv.)

Examination, November-2019

(2016 Admission Onwards)

BHM 502: ADVANCED COMPLEX ANALYSIS

Time: 3 Hours

Max. Marks: 60

SECTION - A

Answer any 4 out of 5 questions. Each question carries 1 Mark. (4×1=4)

- 1. Evaluate $\int_{0}^{\pi/6} e^{i2t} dt$.
- 2. Write Cauchy's integral formula.
- 3. Write the Laurent series expansion of $e^{\frac{1}{z}}$.
- **4.** Find the order of the pole at z = 0 of the function $f(z) = \frac{\sinh z}{z^4}$.
- State Rouche's theorem.

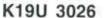
SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 Marks.

(6×2=12)

6. Suppose $c = \{z: z = 2e^{i\theta}, 0 \le \theta \le \frac{\pi}{2}\}$ with positive orientation. Show that $\left| \int_{c} \frac{z+4}{z^3-1} dz \right| \le \frac{6\pi}{7}.$

P.T.O.





7. If a function f is analytic throughout a simply connected domain D, then prove that $\int_{c}^{f(z)dz=0}$ for any contour C lying in D.

(2)

- **8.** Evaluate $\int_{c}^{ze^{-z}dz}$ where C is the circle |z|=1
- Prove that absolute convergence of a series of complex numbers implies the convergence of that series.
- 10. Represent the function $f(z) = \frac{z+1}{z-1}$ by maclaurin series.
- 11. Find the order of the zero at z = 0 of $f(z) = z^2(\cos z 1)$.
- 12. Let C denote the circle |z|=1 with positive orientation. Find $\Delta_c \arg f(z)$ where $f(z)=\frac{z^3+2}{z}$.
- 13. State Argument principle.
- **14.** Find the residue of $f(z) = \frac{z+1}{(z-1)^3(z-2)}$ at z=2.

SECTION - C

Answer any 8 questions out of 12 questions. Each question carries 4 Marks.
(8×4=32)

- **15.** Evaluate $\int_{c}^{Z^{a-1}dZ}$ where C is the positively oriented circle $\{z: z=Re^{iv}, -\pi \le \theta \le \pi\}$
- **16.** Suppose that a function f is analytic inside and on a positively oriented circle C_R centered at z_0 with radius R. if M_R in the maximum value of |f(z)| on C_R , then prove that $\left|f_{(z_0)}^{(n)}\right| \leq \frac{n! M_R}{R^n}$, n = 1, 2, 3, ...



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17. Find $\int_{c}^{1} \frac{1}{(z^2+4)^2} dz$ where C is the positively oriented circle |z-i|=2.

(3)

- **18.** Show that if $|z-i| < \sqrt{2}$, $\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}$.
- 19. Find the Laurent series of $f(z) = \frac{1}{(z-1)(z-2)}$ in 1 < |z| < 2.
- **20.** If z_1 is a point inside the circle of convergence $|z-z_0|=R$ of a power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ then prove that the series must be uniformly convergent in the closed disk $|z-z_0| \le R_1$ where $R_1 = |z_0-z_1|$.
- 21. If a function f is analytic every where in the infinite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then prove that $\int_{c} f(z)dz = 2\pi i \mathop{\rm Res}_{z=0}^{\rm Res} \left[\frac{1}{z^2} f\left(\frac{1}{z}\right) \right].$
- **22.** Suppose $f(z) = \frac{(\log z)^3}{z^2 + 1}$ where $\log z = \log r + i\theta$, $0 < \theta < 2\pi, r > 0$. Find the residues at the poles.
- 23. Evaluate $\int_{0}^{\infty} \frac{dx}{x^2 + 1}$ using residues.
- **24.** Evaluate $\int_{0}^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$
- **25.** Determine the number of zeros of $z^4 + 3z^3 + 6$. inside the circle |z| = 2.
- 26. State and prove Residue theorem.

P.T.O.