Answer any **one** essay question out of 2 : **(1×10=10)**

34. State and prove the inverse function theorem.

35. If D is open in $\mathbb{R}^2$, the function $F : D \to \mathbb{R}$ has continuous partial derivatives F_1 and F_2 on D and $(x_0, y_0) \in D$ be such that $F(x_0, y_0) = 0$ and $F_2(x_0, y_0) \neq 0$, show that there is an open interval $I_0 \in \mathbb{R}$ and a continuously differentiable function $\phi : I_0 \to \mathbb{R}$ such that $x_0 \in I_0$, $(x, \phi(x)) \in D$ for all $x \in I_0$, $\phi(x_0) = y_0$ and such that $F(x, \phi(x)) = 0$ for all $x \in I_0$. Also, prove that the formula $\phi'(x) = \frac{-F_1(x, \phi(x))}{F_2(x, \phi(x))}$ is valid for all $x \in I_0$.

Reg. No. : ....

Name : ....

IV Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.) Examination, May 2017
(2013 Admission)
BHM 401 : REAL ANALYSIS – II

Time : 3 Hours Max. Marks : 80

Answer **all** the **10** questions : **(10×1=10)**

1. Define a tagged partition of a closed interval I.
2. If $f \in R[a, b]$, define the indefinite integral of f with base point 'a'.
3. State the additivity theorem in Riemann integration.
4. Give the Cauchy criterion for the Riemann integrability of a function $f : [a, b] \to \mathbb{R}$.
5. Find the indefinite integral of the Thomae's function.
6. Define the n^{th} trapezoidal approximation of a real valued function f.
7. State Dini's theorem.
8. If $f : D \to \mathbb{R}$, where $D \subseteq \mathbb{R}^n$, define the partial derivative of f with respect to x_i at x, where $x = (x_1, x_2, ..., x_n) \in D$.
9. If $\varphi : A \to \mathbb{R}$, $A \subseteq \mathbb{R}$ is a function, define the uniform norm of φ on A.
10. Given $z = x^2 + y^3$, where $x = st$, $y = e^{st}$, find $\frac{\partial Z}{\partial S}$.

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. Show that the Dirichlet function defined by $f(x) = 1$, if $x \in [0, 1]$ is rational and $f(x) = 0$ if $x \in [0, 1]$ is irrational, is not Riemann integrable.

12. If $\varphi : [a, b] \to \mathbb{R}$ is a step function, show that $\varphi \in R[a, b]$.

13. Using the substitution theorem, find the value of $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\,dt$.

14. If F, G are differentiable and if $f = F'$ and $g = G'$ belong to R[a, b], show that $\int_a^b fG = FG\Big|_a^b - \int_a^b Fg$.

15. If $f \in R[a, b]$, prove that $|f| \in R[a, b]$ and $\left|\int_a^b f\right| \le \int_a^b |f|$.

16. Show that the sequence (x^n) converges to zero on $(-1, 1)$ but it is not uniformly convergent on $(-1, 1)$.

17. Prove that the limit of a power series is continuous on the interval of convergence.

18. Obtain the Taylor series expansion of $g(x) = e^x$, $x \in \mathbb{R}$, at $c = 0$.

19. Show that $u = x^2 - y^2$ and $v = 2xy$ is analytic in a neighbourhood of origin.

20. State Leibnitz rule for the differentiation under the integral sign and using this, find the derivative of $F(y) = \int_a^b \sin(xy)dx$.

21. Show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at (0, 0).

22. If f is differentiable at $(x_0, y_0) \in \mathbb{R}^2$ and $\bar{u} = (u_1, u_2)$ is any unit vector, show that $D_{\bar{u}}f(x_0, y_0) = f_1(x_0, y_0)u_1 + f_2(x_0, y_0)u_2$.

23. If $f(x, y) = x^2y^3$, $x = 3$, $y = 1$, $\Delta x = \Delta y = 0.01$, find Δf and df.

24. If $f(x, y) = x^2 - y^2 + 2y$, verify whether $f_{12} = f_{21}$.

Answer **any 6** short essay questions out of 9 : **(6×5=30)**

25. Using the definition of Riemann integral of a function prove that $\int_0^3 g = 8$, where $g(x) = 2$ for $0 \le x \le 1$ and $g(x) = 3$ for $1 < x \le 3$.

26. If f, g $\in$ R [a, b], show that $f + g \in$ R [a, b].

27. If f $\in$ R[a, b], show that f is bounded on [a, b].

28. If $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], show that f $\in$ R[a, b].

29. Given f $\in$ R[a, b], prove that the indefinite integral of f is continuous on [a, b].

30. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A if and only if $\|f - f\|_A \to 0$.

31. If $h_n(x) = 2nxe^{-nx^2}$, $x \in [0, 1]$, show that $\int_0^1 \lim h_n(x)\,dx \ne \lim \int_0^1 h_n(x)\,dx$.

32. If $f(x, y) = x^2 + xy + 2$ and $\bar{u} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, find $D_{\bar{u}}f(1, 1)$.

33. If the sequence (f_n) is a sequence of bounded function on $A \subseteq R$ that converges uniformly to f, show that (f_n) is a Cauchy sequence.