Reg. No. : ....

Name : ....

K19U 0780

IV Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2019

(2013-2015 Admissions)

BHM 401 : REAL ANALYSIS – II

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Define Riemann integrable function.
2. State boundedness theorem.
3. Give an example of a function which is not Riemann integrable.
4. State additivity theorem in Riemann integration.
5. Define pointwise convergence of sequence of functions.
6. Give an example of uniformly convergent sequence of function.
7. State Mean Value Theorem.
8. For the harmonic function $f(x, y) = e^x \cos y$, verify that $f_{12} = f_{21}$.
9. Give an example of an elliptic integral of the second kind.
10. Write the sufficient conditions for differentiability of a function f at a point x_0.

Answer **any 10** short answer questions out of **14** : **(10×3=30)**

11. Prove that $f \in \mathcal{R}[a, b]$ and $g \in \mathcal{R}[a, b]$ then $f + g \in \mathcal{R}[a, b]$.
12. Find the norms of the partitions :

    a) $P_1 = (0, 1, 3, 4)$; b) $P_2 = (0, 2, 3, 4)$ of the interval [0, 4].

P.T.O.

13. State Squeeze theorem.

14. If $f \in \mathcal{R}[a, b]$ and if $[c, d] \subseteq [a, b]$, the restriction of f to [c, d] is in $\mathcal{R}[a, b]$.

15. If $f(x) = \frac{1}{2}x^2, \forall x \in [a, b]$. Find the integral $\int_a^b x\,dx$ by using Fundamental theorem of calculus.

16. State and prove composition theorem.

17. Define uniform norm of a function.

18. Show that $\lim\left(\frac{x}{x+n}\right) = 0$, for all $x \in \mathbb{R}, x \geq 0$.

19. State Cauchy criterion for the series of function.

20. Define Taylor series expansion of $f(x) = e^x$ $x \in \mathbb{R}$ at $c = 0$.

21. Write the partial derivative of the function $f(x, y) = \frac{\sqrt{36 - 4x^2 - y^2}}{3}$.

22. Verify that $f_{12} = f_{21}$ for the function $f(x, y) = e^x \cos y$.

23. If $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is differentiable at (x_0, y_0), show that it is continuous at (x_0, y_0).

24. If $f(x, y) = x^2y^3$, $x = 3$, $y = 1$, $\Delta x = \Delta y = 0.01$, find Δf and df.

Answer **any 6** short answer questions out of **9** : **(6×5=30)**

25. If $f : [a, b] \rightarrow \mathbb{R}$ is continuous, then show that $f \in \mathcal{R}[a, b]$.

26. Prove that if $f(x) = \text{sgn } x$ on $[-1, 1]$, then $f \in \mathcal{R}[a, b]$.

27. Let F, G be differentiable on [a, b] and let $f = F'$ and $g = G'$, belong to $\mathcal{R}[a, b]$. Then, prove that $\int_a^b fG = FG\int_a^b - \int_a^b Fg$.

28. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$.

29. Let (f_n) be a sequence of continuous on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Then prove that f is continuous on A.

30. State and prove Cauchy-Hadamard theorem.

31. Prove that if f is continuous on $[a, b] \times [c, d]$ and $F(y) = \int_a^b f(x, y)\,dx$, then F is continuous on [c, d].

32. Show that $f(x, y) = \begin{cases} \frac{x^3 - xy^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$ is continuous and has first order partial derivatives on $\mathbb{R}^2$, but not differentiable at (0, 0).

33. State and prove Mean Value theorem.

Answer **any one** essay question out of **2** : **(1×10=10)**

34. State and prove first form of fundamental theorem of Calculus.

35. Let f be defined in a neighbourhood of $(x_0, y_0) \in \mathbb{R}^2$. Suppose f has partial derivatives, f_1, f_2, f_{12} and f_{21} in this neighbourhood and that the mixed partials f_{12} and f_{21} are continuous at (x_0, y_0). Prove that $f_{12}(x_0, y_0) = f_{21}(x_0, y_0)$.

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