K16U 1335

Reg. No. : ....

Name : ....

IV Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improve.) Examination, May 2016

BHM 401 : REAL ANALYSIS – II

Time : 3 Hours

Max. Marks : 80

Answer **all** the **10** questions. **(10×1=10)**

1. Define the mesh of a partition P of an interval [a, b].
2. Define the Riemann integral of a function $f : [a, b] \to \mathbb{R}$
3. State the squeeze theorem for a function $f : [a, b] \to \mathbb{R}$, regarding Riemann integration of f.
4. Give the Cauchy criterion for the Riemann integrability of a function $f : [a, b] \to \mathbb{R}$.
5. State the fundamental theorem of integral calculus (second form).
6. Define the mid point approximation of a real valued function f.
7. If $A \subseteq \mathbb{R}$, $\phi : A \to \mathbb{R}$ is a function, define the uniform norm of ϕ on A.
8. Define the Taylor expansion of a real valued function f at $c \in \mathbb{R}$.
9. If $f : \mathbb{R}^2 \to \mathbb{R}$, define the differentiability of f at $(x_0\ y_0)$.
10. Find $\frac{dz}{dt}$ if $z = xy$, where $x = t^2$, $y = t^3$.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. If f and g are in R [a, b] and if $f(x) \le g(x)$ for all $x \in [a, b]$, show that $\int_a^b f \le \int_a^b g$.
12. If J is a subinterval of [a, b] having end points c < d and if $\varphi_j(x) = 1$ for $x \in J$ and $\varphi_j(x) = 0$ elsewhere in [a, b], show that $\varphi_j \in R\,[a,b]$.

P.T.O.

13. If $f \in R[a, b]$, α, β, γ are any numbers and if any two of the following integrals exist, show that the third also exists and $\int_\alpha^\beta f = \int_\alpha^\gamma f + \int_\gamma^\beta f$.

14. If f, $g \in R[a, b]$, show that $f.g \in R[a, b]$.

15. If $f \in R[a, b]$, prove that $|f| \in R[a, b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

16. Show that the sequence $\left(\frac{x}{n}\right)$ converge to zero, but it is not uniformly convergent on $\mathbb{R}$.

17. If R is the radius of convergence of $\sum a_n x^n$ and if K is a closed, bounded interval contained in the interval of convergence (– R, R), show that the power series converges uniformly on K.

18. Obtain the Taylor expansion of $g(x) = e^x$, $x \in \mathbb{R}$ at $c = 0$.

19. If $u(x, y) = y^3 - 3x^2y$ and $v(x, y) = x^3 - 3xy^2$, show that u and v satisfy Cauchy-Riemann equations.

20. If $f(x, y) = x^2 - y^2 + 2y$, verify whether $f_{12} = f_{21}$.

21. State Leibnitz rule for differentiation under the integral sign and using this find the derivative of $F(y) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-y^2 \sin x}}$.

22. If $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable at (x_0, y_0), show that it is continuous at (x_0, y_0).

23. If $f(x, y) = x^2y^3$, $x = 3$, $y = 1$, $\Delta x = \Delta y = 0.01$, find Δf and df.

24. Prove that any function of the form $f(x, t) = g(x+ct)$, satisfies $c^2 \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial t^2}$, where c is a constant and f(x, t) describes the vertical displacement of a particle in a wave corresponding to horizontal coordinate x at time t.

Answer **any 6** short essay questions out of **9**. **(6×5=30)**

25. If $f \in R[a, b]$, show that the Riemann integral of f is uniquely determined.

26. If $F(x) = 1$ for $x = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$ and $F(x) = 0$ elsewhere on [0, 1], show that $F \in R[0, 1]$ and $\int_0^1 F = 0$.

27. If $f \in R[a, b]$, show that f is bounded on [a, b].

28. If $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], show that $f \in R[a, b]$.

29. Given $f, F : [a, b] \to \mathbb{R}$, E is a finite set in [a, b], F is continuous on [a, b], $F^1(x) = f(x)$ for all $x \in [a, b] \setminus E$ and $f \in R[a, b]$, show that $\int_a^b f = F(b) - F(a)$.

30. If the sequence (f_n) is a sequence of bounded functions on $A \subseteq \mathbb{R}$ that converges uniformly to f, show that (f_n) is a Cauchy sequence.

31. If (f_n) is a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$ show that f is continuous.

32. Given $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(x, y) = 0$ if $(x, y) = (0, 0)$, show that $f_{12}(0, 0) \neq f_{21}(0, 0)$.

33. If f is continuous on $R = [a, b] \times [c, d]$ and $F(y) = \int_a^b f(x, y)dx$, show that F is continuous on [c, d].

Answer **any one** essay question out of **2**. **(1×10=10)**

34. If $f \in R[a, b]$, f is continuous at a point $c \in [a, b]$ and if F is the indefinite integral of f, show that F is differentiable at c and $F'(c) = f(c)$.

35. State and prove the inverse function theorem.
