K18U 0310

Reg. No. : ....

Name : ....

IV Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.) Examination, May 2018
BHM 402 : ADVANCED ABSTRACT ALGEBRA
(2016 Admission Onwards)

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of **5** questions. **Each** question carries **1** mark.

1. Write the left cosets of the subgroup $3\mathbb{Z}$ of $\mathbb{Z}$.
2. State true or false : Every factor group of a cyclic group is cyclic.
3. Find the value of (12) (16) in the ring $\mathbb{Z}_{24}$.
4. Give example for an integral domain which is not a field.
5. State true of false : $x^2 - 3$ is irreducible over $\mathbb{Q}$. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks.

6. What do you mean by index of a subgroup in a group ?
7. Define group homomorphism.
8. What do you mean by a factor group ?
9. Define a simple group. Give an example.
10. Define a ring. Give an example for a non-commutative ring.

11. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.

12. What is the reminder of 8^{103} when divided by 13.

13. Find all zeros of $x^5 + 3x^3 + x^2 + 2x$ in $\mathbb{Z}_5$.

14. State factor theorem. Factorize $x^4 + 4$ in $\mathbb{Z}_5[x]$. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. State and prove Lagranges theorem.

16. Let ϕ be a homomorphism of a group G into a group G'. Prove that if e is the identity element in G, then $\phi(e)$ is the identity element in G'.

17. Prove that a group homomorphism $\phi : G \to G''$ is a one-to-one amp if and only if $\mathrm{Ker}(\phi) = \{e\}$.

18. Show that if H and N are subgroups of a group G and N is normal in G, then $H \cap N$ is normal in H. Show by an example that $H \cap N$ need not be normal in G.

19. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.

20. Show that the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_6) / \langle(2,3)\rangle$ is isomorphic to $\mathbb{Z}_4 \times \mathbb{Z}_3$ or $\mathbb{Z}_{12}$.

21. Prove that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

22. State and prove Euler's theorem.

23. Find all solutions of the congruence equation $15x \equiv 27 \pmod{18}$.

24. Prove that the set R[x] of all polynomials in an indeterminate x with coefficients in a ring R is a ring under polynomial addition and multiplication.

25. Discuss the irreducibility of $f(x) = x^4 - 2x^2 + 8x + 1$ over $\mathbb{Q}$.

26. State and prove Eisenstein Criterion for irreducibility of a polynomial. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. a) Let H be a subgroup of a group G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Prove that $\sim_L$ is an equivalence relation on G.

 b) Prove that every group of prime order is cyclic.

28. State and prove fundamental homomorphism theorem.

29. a) Prove that every finite integral domain is a field.

 b) Show that for every integer n, the number $n^{33} - n$ is divisible by 15.

30. State and prove division algorithm for F[x]. **(2×6=12)**