Answer **any one** essay questions out of **2**. **(1×10=10)**

34. Let T be a linear operator on a finite-dimensional vector space V. Prove that T is diagonalizable if and only if the minimal polynomial is of the form $p(t)=(t-\lambda_1)(t-\lambda_2).....(t-\lambda_k)$ where $\lambda_1, \lambda_2........\lambda_k$, are the distinct eigen values of T.

35. Find the square root of the matrix $A=\begin{bmatrix} 6 & 2 & -2 \\ 2 & 6 & -2 \\ -2 & -2 & 10 \end{bmatrix}$.

Reg. No. : ....

Name : ....

IV Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, May 2016
BHM 404 : LINEAR ALGEBRA – II

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Let $g : R^3 \to F$ defined by $g(a_1, a_2, a_3) = a_1 - 2a_2 + 4a_3$. Find a vector y such that $g(x, y) = \langle x, y\rangle$, $\forall x \in R^3$.
2. What do you mean by a self-adjoint operator ?
3. What do you mean by orthogonal projection ?
4. Give an example for a bilinear transformation.
5. What do you mean by a symmetric bilinear form ?
6. Find the minimal polynomial of $A=\begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix}$
7. What do you mean by a generalized eigen space ?
8. Define a matrix norm.
9. Define spectral radius of a matrix.
10. What do you mean by Rayleigh quotient of a matrix ?

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Let V be a finite-dimensional inner product space over F and let β be an orthonormal basis for V. Prove that if T is a linear operator on V, then $[T^*]_\beta = [T]_\beta^*$.
12. Let T be a linear operator on a finite-dimensional inner product space V. Prove that if T has an eigen vector, then T* has also an eigen vector.

13. Give an example to show that complex symmetric matrices need not be normal.

14. Define a unitary operator. Let $h \in H$ satisfy $|h(x)| = 1$ for all x. Define the linear operator T on H by T(f) = hf. Show that T is unitary.

15. Prove that the minimal polynomial p(t) of a linear operator T on a finite dimensional vector space V divides any other polynomial g(t) with $g(T) = T_0$.

16. Let $H : R^2 \times R^2 \to R^2$ be the function defined by $H((a_1, a_2), (b_1, b_2)) = a_1 b_2 - b_1 a_2$ for $(a_1, a_2), (b_1, b_2) \in R^2$. Prove that H is a bilinear form. Also find a 2×2 matrix A such that $H(x, y) = x^t Ay$ for all $x, y \in R^2$.

17. Let T be a linear operator on a vector space V and let λ be an eigen value of T. Prove that for any scalar $\mu \neq \lambda$, the restriction of $T - \mu I$ to K_λ is one-to-one.

18. Define a reflection operator. Let $T : R^2 \to R^2$ be defined by T(a, b) = (–a, b) and let W = span $(\{e_1\})$. Show that T is a reflection on R^2 about $W^\perp$.

19. Using Gauss Jordan method, solve 2x + y + z = 10, 3x + 2y +3z = 18, x + 4y + 9z = 16.

20. Factorize the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ into the LU form.

21. If $X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $Y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ and $W = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ find the distance between X and Y with respect to the Euclidean norm and the inner product norm with respect to W.

22. Determine whether the matrix $A = \begin{bmatrix} 2 & -17 & 7 \\ -17 & -4 & 1 \\ 7 & 1 & -14 \end{bmatrix}$ is positive definite.

23. Prove that all eigen values of a unitary matrix have absolute value equal to one.

24. Prove that the product of unitary matrices of the same order is also a unitary matrix.

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. Let $A \in M_{m \times n}(F)$ and $b \in F^m$. Suppose that Ax = b is consistent. Prove that there exists exactly one minimal solution s of Ax = b and $s \in R(L_{A^*})$.

26. Let T be a linear operator on a finite dimensional complex inner product space V. Prove that T is normal then there exists an orthonormal basis for V consisting of eigen vectors of T.

27. Define a symmetric bilinear form. Prove that a bilinear form H on a finite dimensional vector space V is symmetric if and only if $\psi_\beta(H)$ is symmetric where, β is an ordered basis for V.

28. Prove that every symmetric bilinear form on a finite-dimensional vector space over a field is diagonalizable if the characteristic of the field is not equal to two.

29. Let T be a linear operator on an n-dimensional vector space V such that V is a T-cyclic subspace of itself. Prove that the characteristic polynomial and minimal polynomial have the same degree.

30. Using Gauss method, find the inverse of $A = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$

31. Solve the equations 5x – 2y + z = 4, 7x + y – 5z = 8, 3x + 7y + 4z = 10 by factorization method.

32. Calculate the Frobenius norm, L_1 norm, L_∞ norm and spectral norm for

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -4 & -6 & 0 \\ 0 & 0 & -9 \end{bmatrix}.$$

33. What do you mean by Schur decomposition of a matrix ? Explain an algorithm for producing Schur decomposition for an $n \times n$ matrix.