Answer **any one** essay question out of 2 : **(1×10=10)**

34. Let T be a linear operator on a finite dimensional vector space V. Prove that T is diagonalisable if and only if the minimal polynomial is of the form $p(t) = (t - \lambda_1)(t - \lambda_2) \dots (t - \lambda_n)$, where $\lambda_1, \lambda_2, \dots, \lambda_n$ are the distinct eigen values of T.

35. Determine the inertia matrix of $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 3 & 4 \end{pmatrix}$.

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**Reg. No. :** ...............................

**Name :** ...................................

# IV Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2019 (2013-2015 Admissions)
## BHM 404 : LINEAR ALGEBRA – II

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Let $g : R^2 \to RF$ defined by $g(a_1, a_2) = 2a_1 + a_2$. Find a vector y such $g(x) = \langle x, y \rangle$ for all $x \in R^2$.

2. If $T^*$ is the adjoint of a linear operator T, then show that $T^*$ is linear.

3. State Spectral Theorem.

4. Define symmetric and skew symmetric bilinear form.

5. Define an orthogonal projection in a inner product space.

6. Let T be a linear operator on $R^2$ defined by $T(a, b) = (2a + 5b, 6a + b)$. Find the minimal polynomial of T.

7. Distinguish between characteristic polynomial and minimal polynomial of a square matrix.

8. Define indefinite matrix.

9. Define spectral norm.

10. Find the matrix form of the quadratic form $3x_1^2 - 2x_1x_2 + x_2^2 + 3x_3 - 2x_2x_3$.

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. If $T^*$ is the adjoint of a linear operator T, then prove that $(T^*)^* = T$.

12. Let T be a self-adjoint operator on a finite-dimensional inner product space V. Then prove that every eigen value of T is real.

13. Let T be a linear operator on a finite-dimensional real inner product space V. Show that T is self-adjoint if and only if there exists an orthonormal basis P of eigenvectors of T.

14. Let $T : R^3 \to R^2$ given by $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 + 3x_3 \\ 4x_1 + 5x_2 + 6x_3 \end{pmatrix}$, then, find $T^* : R^2 \to R^3$.

15. Show that an $n \times n$ matrix A is unitary diagonalizable if and only if A is normal.

16. Show that the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, is Hermitian, and so unitary diagonalisable.

17. Prove that the minimal polynomial of a linear operator on a finite dimensional vector space is unique.

18. Let $H : R^2 \times R^2 \to R^2$ be the function defined by $H((a_1, a_2), (b_1, b_2)) = 2a_1b_1 + 3a_1b_2 + 4a_2b_1 - a_2b_2$ for $(a_1, a_2), (b_1, b_2) \in R^2$. Find the matrix representation of H with respect to $\beta = \{(1, 1), (1, -1)\}$.

19. Using Gauss elimination method, solve $x + y + z = 3$, $x + 2y + 2z = 5$, $3x + 4y + 4z = 11$.

20. Normalize the vector $X = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, Y = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, with respect to $l_1$ and $l_2$ norm.

21. Factorize the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ into the LU form.



22. Find the elementary reflector associated with $V = [9, 3, -6]^T$.

23. Determine whether the matrix $A = \begin{pmatrix} 6 & 2 & -2 \\ 2 & 6 & -2 \\ -2 & -2 & 10 \end{pmatrix}$, is positive definite.

24. Show that $\lambda$ if is an eigen value of a unitary matrix, then $|\lambda| = 1$.

Answer **any 6** short answer questions out of **9** : **(6×5=30)**

25. Find the minimal solution of $x + 2y - z = 12$.

26. Using Gauss method, find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

27. Let $H : \mathbb{F}^2 \times \mathbb{F}^2 \to \mathbb{Z}_2$ be the function defined by $H((a_1, a_2), (b_1, b_2)) = a_1b_1 + a_2b_1$. Check whether H is diagonalizabe or not.

28. Find the Jordan Canonical form for $A = \begin{pmatrix} 0 & 1 & -1 \\ -4 & 4 & -2 \\ -2 & 1 & 1 \end{pmatrix}$.

29. Prove that a cycle of generalized eigen vectors of a linear operator on a vector space corresponding to an eigen value is linearly independent.

30. Compute minimal polynomial for the matrix $A = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 2 & 2 \\ -1 & 0 & 1 \end{pmatrix}$.

31. Solve the equations $5x - 2y + z = 4$, $7x + y - 5z = 8$, $3x + 7y + 4z = 10$ by factorization method.

32. Check for diagonalization of the matrix $A = \begin{pmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{pmatrix}$.

33. Find the spectrum of the matrix $A = \begin{pmatrix} 7 & 8 & 4 \\ -3 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix}$.