Answer **any one** : **(1×10=10)**

34. The data given below gives the miles per gallon obtained by a test automobile when using gasolines of varying octane levels.

Miles per Gallon (y)	Octane (x)
13.0	89
13.2	93
13.0	87
13.6	90
13.3	89
13.8	95
14.1	100
14.0	98

i) Calculate the value of r

ii) Do the data provide sufficient evidence to indicate that octane level and miles per gallon are dependent. Give the attained significance level, and indicate your conclusion if you wish to implement an $\alpha = 0.05$ level test.

35. Twelve overweight subjects participated in a study comparing the effectiveness of three weight-reducing diets. The subjects were grouped according to their initial weight, and each of three subjects from each initial weight group was randomly assigned to a diet. The weight loss (in pounds) at the end of experimental period is given below :

	Diet		
Initial Weight	**A**	**B**	**C**
150 – 174	10	23	24
175 – 199	12	21	26
200 – 224	12	31	21
Over 224	20	28	33

i) Do these data provide sufficient evidence that (after eliminating the effect of initial weight) the diets are different in their effectiveness ?

ii) Does the initial weight affect the loss of weight ? ($\alpha = 0.01$).

Reg. No. : ....

Name : ....

IV Semester B.Sc. (Hon's) Degree (Mathematics – (Regular)) Examination, May 2015
BHM 405 : ADVANCED STATISTICS – II

Time : 3 Hours

Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Define deterministic models with example.
2. If $\hat{\beta}_1 > 0$ what type of correlation is there.
3. Define coefficient of determination.
4. State Bonferroni inequality.
5. What is factor ?
6. Describe categorical data.
7. Describe randomization in design of experiments.
8. Describe completely randomized design.
9. Give the 100 (1 – α)% confidence interval for β_1.
10. The correlation coefficient for heights and weights of ten offensive back field football players was determined to be $\gamma = 0.826$. What percentage of the variation in weights was explained by the height of the players.

Answer **any 10** short answer questions : **(10×3=30)**

11. Give the properties of least square estimates in a simple linear regression model.
12. S.T. the least square equation $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ always goes through the point $(\bar{x}, \bar{y})$ if $\hat{\beta}_0$ and $\hat{\beta}_1$ are least square estimates for the intercept and slope in a simple linear regression model.
13. Give the test procedure for testing the hypothesis for β_i.
14. State the assumptions underlying the ANOVA for a randomized block design.

15. Give any three applications of χ^2 test.
16. Give the statistical model for one-way layout.
17. Suppose that the model under consideration is simple linear regression model based on normal theory. In order to test $H_0 : \beta_1 = 0$ the test statistic used is
$T = \frac{\hat{\beta}_1 - 0}{S/\sqrt{Sxx}}$ which possesses t distribution with n – 2 d.f. S.T. the equation for T can also be written as $T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$.
18. Suppose that we wish to compare the means for two populations and that $\sigma_1^2 = 9, \sigma_2^2 = 25$ and n = 90. What allocation of n = 90 to the two samples will result in the maximum amount of information about $(\mu_1 - \mu_2)$?
19. Refer to the statistical model for one-way layout S.T. $H_0 : T_1 = T_2 = ... = T_k = 0$ is equivalent to $H_0 : \mu_1 = \mu_2 = ... = \mu_k$.
20. Explain Chi-square test for goodness of fit.
21. Give the 100 $(1-\alpha)$% prediction interval for Y when $x_1 = x_1^*, x_2 = x_2^*, ..., x_k = x_k^*$.
22. Distinguish between deterministic and probabilistic models with examples.
23. Consider the model $Y_i = \beta_1 x_i + \epsilon_i$; i = 1, 2 ...n where ϵ_i's are independently and identically distributed random variables with E (ϵ_i) = 0. Find the least square estimator of β_1.
24. Suppose that the entry in the i^{th} row and j^{th} column in a r × c contingency table is nij for i = 1, 2 ... r and j = 1, 2, ... C. Let the row and column totals are denoted by r_i for i = 1, 2... r and Cj for j = 1, 2 .... C and the total sample size is n.

$$\text{S.T. } \chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{[n_{ij} - \widehat{E(n_{ij})}]^2}{\widehat{E(n_{ij})}} = n\left[\sum_{i=1}^{r}\sum_{j=1}^{c} \frac{n_{ij}^2}{r_i C_j} - 1\right].$$

Answer **any 6** out of 9 : **(6×5=30)**

25. Fit a straight line to the five data points in the following table. Give the estimates of β_0 and β_1.

y	3.0	2.0	1.0	1.0	0.5
x	–2.0	–1.0	0	1.0	2.0

26. S.T. $\hat{\beta}_1$ is an unbiased estimator for β_1.
27. Give the analysis of variance for a randomized block design.
28. A completely randomized design is to be conducted to compare five teaching techniques in classes of equal size. Estimation of the differences in mean response on an achievement test is desired correct to within 30 test-score points, with probability equal to 0.95. It is expected that the test scores for a given teaching technique will possess a range approximately equal to 240. Find the approximate number of observations required for each sample in order to acquire the specified information.
29. The reaction times for two different stimulus in a psychological word-association experiment were compared by using each stimulus on independent random samples of size 8. Thus a total of 16 people were used in the experiment. Do the following data present sufficient evidence to indicate that there is a difference in the mean reaction times for the stimuli ?

Stimulus – 1	1	3	2	1	2	1	3	2
Stimulus – 2	4	2	3	3	1	2	3	3

30. Let $\overline{Y}i.$ denote the average of all the responses to treatment i. Use the model for the randomized block design to derive $E(\overline{Y}i.)$ and $V(\overline{Y}i.)$.
31. Describe the independent multinomial populations whose proportions are compared in the Chi-square analysis.
32. Suppose $Y_1, Y_2, ... Y_n$ are independent normal random variables with $E(Yi) = \beta_0 + \beta_1 x_i$ and $v(Yi) = \sigma^2$ for i = 1, 2 ... n. S.T. the MLE's of β_0 and β_1 are the same as the least-square estimators.
33. Consider the general linear model
$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \epsilon$ where $E(\epsilon) = 0$ and $V(\epsilon) = \sigma^2$. Let $\hat{\beta}_i = a'\hat{\beta}$ where the vector a is defined by
$$a_j = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases}$$
Verify that $E(\hat{\beta}_i) = \beta_i$ and $v(\hat{\beta}_i) = C_{ii}\,\sigma^2$ where C_{ii} is the element in the i^{th} row and j^{th} column of $(X'X)^{-1}$.