Reg. No. : ....

Name : ....

K19U 0765

IV Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supp./Imp.)
Examination, April 2019
(2016 Admission Onwards)
BHM 401 : ADVANCED REAL ANALYSIS AND METRIC SPACES

Time : 3 Hours | Total Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define a tagged partition. Give an example.
2. State first form of fundamental theorem.
3. Find $\lim \left(\frac{x^2 + nx}{n}\right)$ for $x \in \mathbb{R}$.
4. Define complete metric space.
5. Let (X, d) be a metric space. Define the terms :
   a) Open ball
   b) Closed ball
   c) Neighbourhood of $x_0 \in X$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.**(6×2=12)**

6. Let $f : [0, 4] \rightarrow \mathbb{R}$ be defined by $f(x) = 2$ for $0 \leq x \leq 3$ and $f(x) = 1$ for $3 < x \leq 4$. Find $\int_0^4 f dx$.
7. Let $h(x) = x$ for $x \in [0, 2]$. Show that $h \in \mathfrak{R}[0, 2]$.
8. If $f, g \in \mathfrak{R}[a, b]$, then show that $f + g \in \mathfrak{R}[a, b]$.

P.T.O.

9. Define radius of convergence of a power series. Illustrate it.

10. If J is a subinterval of [a, b] having end points c < d and $\phi(x) = 1$ for $x \in J$ and $\phi(x) = 0$ elsewhere in [a, b]. Then show that $\phi \in \mathfrak{R}[a, b]$ and $\int_a^b \phi dx = d - c$.

11. Prove that a convergent sequence in a metric space is a Cauchy sequence.

12. Let (x, d) be a metric space. Define $d' : X \times X \to \mathbb{R}$ by $d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}$. Then prove that d' is a metric on X.

13. Show that in any metric space (X, d) each open ball is an open set.

14. Let F be a nonempty bounded closed subset of $\mathbb{R}$ and let $\alpha = \inf F$ and $\beta = \sup F$. Then show that $\alpha \in F$ and $\beta \in F$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. If $f \in \mathfrak{R}[a, b]$ then show that f is bounded on [a, b].

16. State and prove Squeeze theorem.

17. If $f : [a, b] \to \mathbb{R}$ is continuous on [a, b] then show that $f \in \mathfrak{R}[a, b]$.

18. State and prove Cauchy Criterion for uniform convergence of sequence of functions.

19. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$ then show that f is continuous on A.

20. If R is the radius of convergence of the power series $\sum a_n x^n$, then show that the series is absolutely convergent if $|x| < R$ and is divergent if $|x| > R$.

21. State and prove Holder's inequality.

22. If a Cauchy sequence of points in a metric space (X, d) contains a convergent subsequence, then prove that the sequence converges to the same limit as the sequence.

23. Define the following :

a) Discrete metric on a set X.

b) Usual metric on $\mathbb{R}$.

c) Euclidean metric on $\mathbb{R}^n$.

d) Pseudometric on a set X.

e) Square summable sequences.

f) The space of all bounded sequences.

g) The space of all bounded functions.

h) Cauchy sequence.

24. Let Y be a subspace of a metric space (X, d), then show that every subset of Y that is open in Y is also open in X if and only if Y is open in X.

25. Show that a subset G in a metric space (X, d) is open if and only if it is the union of all open balls contained in G.

26. Let (X, d) be a metric space and F be a subset of X, then show that F is closed in X if and only if F^c is open in X.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. When we say that a function $f : [a, b] \to \mathbb{R}$ is Riemann integrable on [a, b] ? If $f \in \mathfrak{R}[a, b]$, then show that the value of the integral is uniquely determined.

28. a) Prove that the function $f(x) = \frac{x}{1+x}$ is monotonically increasing.

b) If $f : [a, b] \to \mathbb{R}$ is monotone on [a, b] then show that $f \in \mathfrak{R}[a, b]$.

29. Show that the l_p space is complete.

30. Let (X, d) be a metric space. Then show that

i) ϕ and X are open sets in (X, d).

ii) the union of any finite, countable or uncountable family of open sets is open.

iii) the intersection of any finite family of open sets is open.
