Reg. No. : ....

Name : ....

K18U 0315

IV Semester B.Sc. Hon's (Mathematics) Degree (Supple./Improv.) Examination, May 2018

BHM 402 : ABSTRACT ALGEBRA – II

(2013-15 Admissions)

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. What do you mean by decomposable group ?

2. What are the elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$?

3. What are the elements of $\mathbb{Z}/5\mathbb{Z}$?

4. State first isomorphism theorem.

5. What do you mean by a subnormal series of a group ?

6. What do you mean by action of a group on a set ?

7. State Burnside's formula.

8. Define a p-group.

9. What is the reduced form of the word $a_2^3 a_2^{-1} a_3 a_1^2 a_1^{-7}$?

10. What do you mean by presentation of a group ?

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Find the order of (8, 4, 10) in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$.

12. What do you mean by normal subgroup ? Write three equivalent conditions for a subgroup H of a group G to be a normal subgroup of G.

13. Prove that a factor group of a cyclic group is cyclic.

P.T.O.

14. What do you mean by simple group ? Give an example.

15. Let $\phi : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_3$ be the homomorphism such that $\phi(1) = 2$. Find kernal of ϕ.

16. Give example for a series which is a subnormal series of the group, but not a normal series.

17. Prove that any two composition series of a group are isomorphic.

18. Let X be a G-set. Prove that G_x is a subgroup of G for each $x \in G$.

19. How many distinguishable necklaces (with no clasp) can be made using seven different colored beads of the same size ?

20. Let G be a group of order p^n, p is a prime, and let X be a finite G-set. Show that $|X| \equiv |X_G| \pmod{p}$.

21. Prove that every group of order 15 is cyclic.

22. Is $\{(3, 0), (0, 1)\}$ a basis for $\mathbb{Z} \times \mathbb{Z}$? Prove your assertion.

23. Explain the idea of free group.

24. Prove that every group G' is a homomorphic image of a free group G.

Answer **any 6** short answer questions out of **9**. **(6×5=30)**

25. Let H be a normal subgroup of a group G. Prove that the cosets of H form a group G/H under the operation (aH) (bH) = abH.

26. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.

27. If N is a normal subgroup of G and if H is any subgroup of G, then prove that $H \vee N = HN = NH$.

28. If G has a composition series and if N is a proper normal subgroup of G, then prove that there exists a composition series containing N.

29. Let X be a G-set. Show that for each $g \in G$, the function $\sigma_g : X \rightarrow X$ defined by $\sigma_g(x) = gx$ for $x \in X$ is a permutation of X.

30. State and prove second Sylow theorem.

31. Prove that for a prime number p, every group G of order p^2 is abelian.

32. Let G be generated by $A = \{a_i / i \in I\}$ and let G' be any group. If a_i' for $i \in I$ are any elements in G', not necessarily distinct, then prove that there is atmost one homomorphism $\phi : G \rightarrow G'$ such that $\phi(a_i) = a_i'$.

33. Show that $(x, y : y^2x = y, yx^2y = x)$ is a presentation of the trivial group of one element.

Answer **any one** essay questions out of **2**. **(1×10=10)**

34. State and prove second and third isomorphism theorems.

35. Determine all groups of order 10 upto isomorphism.