

K19U 3016

(4)



29. Verify the circulation form of Green's theorem on the annular ring

$$R: h^2 \leq x^2 + y^2 \leq 1, 0 < h < 1 \text{ if } M = \frac{-y}{x^2 + y^2}, N = \frac{x}{x^2 + y^2}$$

30. Verify Stokes theorem for $\vec{f} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle

bounded by $x = a, x = -a, y = 0$ and $y = 6$.



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Reg. No. :

000735

Name :

III Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.)

Examination, November-2019

(2016 Admission Onwards)

BHM 302 : VECTOR CALCULUS

Time : 3 Hours

Max. Marks : 60

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark. (4 × 1 = 4)

1. Define torsion of a curve.
2. State Fubini's theorem (First form)
3. Find the gradient field of $f(x, y, z) = xyz$.
4. Write the formula for the workdone by a force \vec{f} to move a particle from a point A to another point B.
5. Check whether the vector $\vec{F} = (2x - 3)\vec{i} - z\vec{j} + \cos z\vec{k}$ is conservative or not.

SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 mark.

(6 × 2 = 12)

6. Explain the idea of limits and continuity of a vector function $\vec{r}(t)$ at a point t_0 .
7. Evaluate $\int (\cos t \vec{i} + \vec{j} - 2t \vec{k}) dt$.
8. Find the length of one turn of the helix $\cos t \vec{i} + \sin t \vec{j} + t \vec{k}$

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9. Calculate $\iint_R (1-6x^2y) dx dy$ where R is $0 \leq x \leq 2, -1 \leq y \leq 1$.
10. Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.
11. A fluid's velocity field is $\vec{f} = xi + zj + yk$. Find the flow field along the helix $\vec{r}(t) = \cos t i + \sin t j + y k, 0 \leq t \leq \frac{\pi}{2}$.
12. Find the workdone by the conservative field $\vec{f} = yzi + xzj + xyk$ along any smooth curve C joining the points $(-1, 3, 9)$ to $(1, 6, -4)$.
13. State Green's theorem.
14. Find curl \vec{f} if $\vec{f} = (x^2 - y)i + 4zj + x^2k$.

SECTION - C

Answer any 8 questions out of 12 questions. Each question carries 4 marks. (8x4=32)

15. The vector $\vec{r}(t) = (3\cos t)i + (3\sin t)j + t^2k$ gives the position of a moving body at time t . Find the body's speed and direction when $t=2$. At what time, if any, are the body's velocity and acceleration orthogonal?
16. Find the unit tangent vector and unit normal vector for the circular motion $\vec{r}(t) = \cos 2t i + \sin 2t j$.
17. Find the derivative of $f(x, y) = x^2 + xy$ at $(1, 2)$ in the direction of the vector $i + j$.
18. Find the volume of the prism whose base is the triangle in the xy plane bounded by the x -axis and the lines $y = x$ and $x = 1$ and whose top lines in the plane $z = 3 - x - y$.
19. Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ boundad by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.



20. Find the volume of the upper region D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$.
21. Integrate $f(x, y, z) = x - 3y^2 + z$ along the line segment from origin to $(1, 1, 0)$ and then to $(1, 1, 1)$.
22. What do you mean by circulation of a vector field around a closed curve? Find the circulation of $\vec{F} = (x - y)i + xj$ around the circle $\vec{r}(t) = (\cos t)i + (\sin t)j, 0 \leq t \leq 2\pi$.
23. Show that $y dx + x dy + 4 dz$ is exact and evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} y dx + x dy + 4 dz$ over the line segment from $(1, 1, 1)$ to $(2, 3, -1)$.
24. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.
25. Find the flux of $\vec{F} = yzj + z^2k$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1, z \geq 0$, by the planes $x = 0$ and $x = 1$.
26. Using Stoke's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$, if $\vec{F} = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counterclockwise as views from above.

SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 marks. (2x 6 = 12)

27. State and prove differentiation rules for dot product and cross product of two vector functions.
28. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x + 2) dy dx$, with respect to the equivalent integral with the order of integration reversed and hence evaluate the integral.

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