



Reg. No. :

Name :

Time : 3 Hours

SECTION - A

1. Define the continuity of a vector function $\mathbf{r}(t)$ at a point $t = t_0$ in its domain.
2. State quotient rule for gradients.
3. State first form of Fubini's theorem.
4. Define the potential function for a vector field \mathbf{F} .
5. Define the curvature function of a curve.

SECTION – B

6. A glider is soaring upward along the helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$. How long is the glider's path from $t = 0$ to $t = 2\pi$.
7. Find \mathbf{T} and \mathbf{N} for the circular motion $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j}$.
8. Find the curvature of a circle.
9. Find the volume of the region bounded above by the elliptical paraboloid $z = 16 - x^2 - y^2$ and below by the square $R: 0 \leq x \leq 2, 0 \leq y \leq 2$.
10. Find the work done by the conservative field $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} = \nabla f$, where $f(x, y, z) = xyz$ along any smooth curve (joining the points $A(-1, 3, 9)$ to $B(1, 6, -4)$).

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11. Find the circulation of the field $F = (x - y) \mathbf{i} + x \mathbf{j}$ around the circle $\mathbf{r}(t) = (\cos t) \mathbf{i} + (\sin t) \mathbf{j}$, $0 \leq t \leq 2\pi$.
12. Integrate $F(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.
13. State Divergence Theorem.
14. State first form of Fubini's theorem.

SECTION - C

(Answer **any 8** questions out of 12 question. **Each** question carries **4** marks.) **(8×4=32)**

15. Find the unit tangent vector of the curve $\mathbf{r}(t) = (3 \cos t) \mathbf{i} + (3 \sin t) \mathbf{j} + t^2 \mathbf{k}$ representing the path of the glider.
16. Find the curvature for the helix $\mathbf{r}(t) = (a \cos t) \mathbf{i} + (a \sin t) \mathbf{j} + (bt) \mathbf{k}$, $a, b > 0$, $a^2 + b^2 \neq 0$.
17. Find the derivative of $f(x, y) = x e^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.
18. Find the area enclosed by the lemniscate $r^2 = 4 \cos 2\theta$.
19. Find the volume of the solid region bounded above by the paraboloid $z = 9 - x^2 - y^2$ and below by the unit circle in the xy -plane.
20. Evaluate $\iint_R e^{x^2+y^2} dy dx$ where R is the region (semicircular) bounded by the X-axis and the curve $y = \sqrt{1-x^2}$.
21. Find the Jacobian for the polar coordinate transformation $x = r \cos \theta$, $y = r \sin \theta$ and find the Cartesian integral $\iint_R f(x, y) dx dy$ as a polar integral.
22. Find the volume of the ice cream cone cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \pi/3$.
23. Integrate $f(x, y) = x - 3y^2 + z$ over the line segment joining $(0, 0, 0)$ to $(1, 1, 1)$.
24. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = z\mathbf{i} + xy\mathbf{j} - y^2\mathbf{k}$ along the curve C given by $\mathbf{r}(t) = t^2 \mathbf{i} + t\mathbf{j} + \sqrt{t}\mathbf{k}$, $0 \leq t \leq 1$.



25. State normal form of Green's theorem and verify for the field $F(x, y) = (x - y) \mathbf{i} + x \mathbf{j}$ and the region bounded by the unit circle.
26. Find the surface area of a sphere of radius 'a'.

SECTION - D

(Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks) **(2×6=12)**

27. Find the tangent line and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $P(1, 2, 4)$.
28. Show that $F = (e^x \cos y + yz) \mathbf{i} + (xz - e^x \sin y) \mathbf{j} + (xy + z) \mathbf{k}$ is a conservative field over its natural domain and find the potential function for it also.
29. Find the center of mass of a thin semi spherical shell of radius 'a' and constant density ' δ '.
30. State and prove cross product rule of vector functions for differentiation.