Reg. No. : ....

Name : ....

K17U 2667

III Semester B.Sc. (Hon's) (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2017
BHM 302 : VECTOR CALCULUS (2013-15 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** the **ten** questions. **(10×1=10)**

1. Find the parametric equations for the line through (3, –4, –1) and parallel to $\vec{v} = \hat{i} + \hat{j} + \hat{k}$.
2. Find the Cartesian equation for the surface $z = r^2$ and identify the surface.
3. What do you mean by the principal unit normal vector for a curve in a plane ?
4. What is the area of a closed and bounded region R in polar form ?
5. What is the volume element in cylindrical coordinates ?
6. Define Jacobian determinant of the coordinate transformation.
7. What do you mean by circulation around a curve ?
8. Write an equivalent statement for a vector $\vec{F}$ to be conservative on D.
9. What is the formula for evaluating the surface area of a surface ?
10. State Stoke's theorem.

SECTION – B

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. Find the equation of the plane through the point (– 3, 0, 7) and perpendicular to $5\hat{i} + 2\hat{j} - \hat{k}$.
12. Find the length of the curve $\vec{r}(t) = (2\cos t)\hat{i} + (2\sin t)\hat{j} + (\sqrt{5}t)\hat{k}, 0 \le t \le \pi$.

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13. Find the curvature of $\vec{r}(t) = (2t+3)\hat{i} + (5-t^2)\hat{j}$.

14. Find the derivative of $f = x^3 + xy$ at (1, 2) in the direction of $\hat{i} + \hat{j}$.

15. Evaluate $\iint_R (1 - 6x^2y)\, dx\, dy$ where R is $0 \le x \le 2, -1 \le y \le 1$.

16. Find the area enclosed by one leaf of the rose $r = 12 \cos 3\theta$.

17. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx\, dy\, dz$.

18. Integrate $f = x - 3y^2 + z$ over the line segment joining the origin and the point (1, 1, 1).

19. Check whether the vector $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is conservative or not.

20. Find a potential function for the vector field
$\vec{F} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$.

21. Define divergence of a vector field. Find the divergence of $\vec{F} = (x^2 + 4y)\hat{i} + (x + y^2)\hat{j}$.

22. Using Green's theorem, find the area of the circle $\vec{r} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}, 0 \le t \le 2\pi$.

23. Find a parametrization of the portion of the cylinder $y^2 + z^2 = 9$ between the planes $x = 0$ and $x = 3$.

24. Prove that div (curl $\vec{f}$) = 0.

SECTION – C

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. What do you mean by quadric surfaces ? Briefly explain different quadric surfaces.

26. The vector $\vec{r} = (3\cos t)\hat{i} + (3\sin t)\hat{j} + t^2\hat{k}$ gives the position of a moving body at time t. Find the body's speed and direction when t = 2. At what time, if any, are the body's velocity and acceleration orthogonal ?

27. Change the order of integration and hence evaluate $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\, dx\, dy$.

28. Find the average value of $F(x, y, z) = x^2 + 9$ over the cube in the first octant bounded by the coordinate planes and the planes x = 2, y = 2 and z = 2.

29. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y - 2x)^2\, dy\, dx$.

30. Find the work done by $\vec{F} = xy\hat{i} + y\hat{j} - yz\hat{k}$ over the curve $\vec{r} = t\hat{i} + t^2\hat{j} + t\hat{k}$, $0 \le t \le 1$.

31. Verify Green's theorem for the field $\vec{F} = -y\hat{i} + x\hat{j}$ over the region bounded by the unit circle $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j}, 0 \le t \le 2\pi$.

32. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane z = 4.

33. Find the surface area of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$.

SECTION – D

Answer **any one** essay questions out of 2. **(1×10=10)**

34. Find the volume of the region in the first octant bounded by the coordinate planes, the plane y + z = 2 and the cylinder $x = 4 - y^2$.

35. Find the center of mass of a thin shell of constant density δ cut form the cone $z = \sqrt{x^2 + y^2}$ by the planes z = 1 and z = 2.