Reg. No. : ....

Name : ....

K16U 2582

III Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2016
BHM 302 : VECTOR CALCULUS

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Find the equation for the plane through (–3, 0, 7) and perpendicular to $\vec{v} = 5\hat{i} + 2\hat{j} - \hat{k}$.
2. What is the relation connecting spherical coordinates and cylindrical coordinates ?
3. Define torsion and binormal vector.
4. What is the area of a closed and bounded region R in polar form ?
5. Define average value of a function in space.
6. Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ of the transformation x = u cos v, y = u sin v.
7. What do you mean by flux of a vector field across a curve ?
8. State Green's theorem in plane.
9. Prove that curl grad f = 0.
10. State Gauss divergence theorem.

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. Show that $\vec{u}(t) = \sin t\hat{i} + \cos t\hat{j} + \sqrt{3}\,\hat{k}$ has constant length and is orthogonal to its derivative.
12. Find the curvature of $\vec{r}(t) = a\cos t\hat{i} + a\sin t\hat{j} + bt\hat{k}$, a, b ≥ 0, $a^2 + b^2 \neq 0$.

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13. What do you mean by directional derivative ? Explain the geometric interpretation of the directional derivative.

14. Find the equation of tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at the point (–2, 1).

15. Integrate f(x, y) = y cos xy over the rectangle $0 \le x \le \pi, 0 \le y \le 1$.

16. Change $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} dydx$ into an equivalent polar integral and hence evaluate it.

17. Evaluate $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz\, dy\, dx$.

18. Evaluate $\int_C (x - y + z - 2)\, ds$ where C is the straight line segment x = t, y = 1 – t, z = 1 from (0, 1, 1) to (1, 0, 1).

19. Show that $\vec{F} = (2x - 3)\hat{i} - z\hat{j} + \cos z\hat{k}$ is not conservative.

20. Find the divergence and curl of $\vec{F} = (x^2 - y)\hat{i} + (xy - y^2)\hat{j}$.

21. Define gradient of a scalar field. Find the gradient field of g(x, y, z) = xy + yz + xz.

22. Using Green's theorem, find the area of the ellipse $\vec{r} = a \cos t\hat{i} + b \sin t\hat{j},\ 0 \le t \le 2\pi$.

23. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9,\ 0 \le z \le 5$.

24. Generalize Green's theorem to three dimensions and explain their relationship to the equations in Stoke's theorem and Divergence theorem.

Answer **any 6** short answer questions out of 9 : **(6×5=30)**

25. Explain hyperboloid of two sheet $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

26. The velocity of a particle moving in space is $\cos t\hat{i} - \sin t\hat{j} + \hat{k}$. Find the particle's position $\vec{r}$ as a function of t if $\vec{r} = 2\hat{i} + \hat{k}$ when t = 0.

27. Change the order of integration and hence evaluate $\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy\, dx$.

28. Find the volume of the upper region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$.

29. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x + y}\, (y - 2x)^2\, dydx$.

30. Find the flux of $\vec{F} = (x - y)\hat{i} + x\hat{j}$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

31. Using Green's theorem, evaluate the integral $\int_C xydy - y^2dx$ where C is the square cut from the first quadrant by the lines x = 1 and y = 1.

32. Find the flux $\vec{F} = yz\hat{j} + z^2\hat{k}$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1,\ z \ge 0$, by the planes x = 0 and x= 1.

33. Integrate g(x, y, z) = x^2 over the cone $z = \sqrt{x^2 + y^2},\ 0 \le z \le 1$.

Answer **any one** essay questions out of 2 : **(1×10=10)**

34. Evaluate :

$$\int_0^3 \int_0^4 \int_{x=y/2}^{x=(y/2)+1} \left(\frac{2x - y}{2} + \frac{z}{3}\right) dx\, dy\, dz$$

by applying the transformation $u = \frac{2x - y}{2},\ v = \frac{y}{2},\ w = \frac{z}{3}$ and integrating over an appropriate region in uvw-space.

35. Find the center of mass of a thin shell of constant density δ cut form the cone $z = \sqrt{x^2 + y^2}$ by the planes z = 1 and z = 2.
