Reg. No.: ....

Name : ....

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K15U 0621

III Semester B.Sc. Hon's (Maths) Degree (Reg./Supple./Improve.) Examination, November 2015

BHM 302 : VECTOR CALCULUS

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Find the parametric equations for the line through (–3, 2, –3) and (1, –1, 4).
2. What is the relation connecting rectangular and cylindrical coordinates ?
3. Find the curvature of a straight line.
4. State Fubini's theorem for calculating double integrals.
5. Define average value of a function in space.
6. Find the Jacobian $\frac{\partial (x, y, z)}{\partial (u, v, w)}$ of the transformation x = u cos v, y = u sin v, z = w.
7. What do you mean by circulation around a curve ?
8. State fundamental theorem of line integral.
9. What do you mean by flux of a three dimensional vector field ?
10. State Stoke's theorem.

Answer **any 10** short answer questions out of **14** : **(10×3=30)**

11. Find the equation of the plane through the points (0, 0, 1), (2, 0, 0) and (0, 3, 0).
12. Find the unit tangent vector of the curve $\vec{r} = (\cos t + t \sin t)\hat{i} + (\sin t - t \cos t)\hat{j}, t > 0$.

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13. Find the plane tangent to the surface $z = x \cos y - ye^x$ at (0, 0, 0).

14. Find the derivative of $f = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of $2\hat{i} - 3\hat{j} + 6\hat{k}$.

15. Integrate $f(x, y) = \frac{1}{xy}$ over the square $1 \le x \le 2, 1 \le y \le 2$.

16. Find the limits of integration for integrating $f(r, \theta)$ over the region R that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

17. Evaluate $\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}(x + y + z)\, dz\, dy\, dx$.

18. Evaluate $\int_C (x + y)\, ds$ where C is the straight line segment $x = t, y = 1 - t, z = 0$ from (0, 1, 0) to (1, 0, 0).

19. Show that $\vec{F} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative.

20. Define curl of a vector field. Find the curl of $\vec{F} = (x^2 - y)\hat{i} + (xy - y^2)\hat{j}$.

21. Find a potential function for the vector field $\vec{F} = (y + z)\hat{i} + (x + z)\hat{j} + (x + y)\hat{k}$.

22. Using Green's theorem, find the area of the circle $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j}, 0 \le t \le 2\pi$.

23. Find a parametrization of the sphere $x^2 + y^2 + z^2 = 1$.

24. Generalize Green's theorem to three dimensions and explain their relationship to the equations in Stoke's theorem and Divergence theorem.

Answer **any 6** short answer questions out of **9**. **(6×5=30)**

25. Explain elliptic cone $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$.

26. The velocity of a particle moving in space is $\cos t\,\hat{i} - \sin t\,\hat{j} + \hat{k}$. Find the particle's position $\vec{r}$ as a function of t if $\vec{r} = 2\hat{i} + \hat{k}$ when t = 0.

27. Calculate $\iint_R \frac{\sin x}{x}\, dA$, where R is the triangle in the xy-plane bounded by the x-axis, the line y = x and the line x = 1.

28. Find the limits of integration in cylindrical coordinates for integration a function $f(r, \theta, z)$ over the region D bounded below by the plane z = 0, laterally by the circular cylinder $x^2 + (y - 1)^2 = 1$, and above the paraboloid $z = x^2 + y^2$.

29. Evaluate $\int_0^4 \int_{x = y/2}^{x = (y/2)+1} \frac{2x - y}{2}\, dx\, dy$ by applying the transformation $u = \frac{2x - y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in uv-space.

30. A fluid's velocity field is $\vec{F} = x\hat{i} + z\hat{j} + y\hat{k}$. Find the flow along the helix $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}, 0 \le t \le \frac{\pi}{2}$.

31. Verify Green's Theorem for the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ and the region bounded by the unit circle $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j}, 0 \le t \le 2\pi$.

32. Integrate $g(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes x = 1, y = 1 and z = 1.

33. Find the surface area of the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.

Answer **any one** essay questions out of **2**. **(1×10=10)**

34. Find the centroid ($\delta = 1$) of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below the xy-plane.

35. What do you mean by differential form and exact differential form ? Show that $ydx + xdy + 4dz$ is exact, and evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$ over the line segment from (1, 1, 1) to (2, 3, –1).