K21U 0231

Reg. No. : ....

Name : ....

III Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2020
(2016 Admission Onwards)
BHM 301 : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. State the Arithmetic – Geometric Mean Inequality.
2. If $S = \left\{1 - {(-1)^n}/{n} : n \in \mathbb{N}\right\}$, find supS.
3. State whether the sequence $((-1)^n n^2)$ is convergent or not.
4. State the Bolzano-Weierstrass theorem for sequences.
5. Give an example to show that 'the product of two uniformly continuous real valued functions need not be uniformly continuous'.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. If $a \in \mathbb{R}$ is such that $0 \le a < \varepsilon$ for every $\varepsilon > 0$, show that $a = 0$.
7. If a set $S \subseteq \mathbb{R}$ contains one of its upper bounds, show that this upper bound is the supremum of S.
8. Prove that $\big||a| - |b|\big| \le |a - b|$, $a, b \in \mathbb{R}$.
9. If $t > 0$, prove that there exists $n_t \in \mathbb{N}$ such that $0 < {1}/{n_t} < t$.
10. Prove that a sequence is $\mathbb{R}$ can have atmost one limit.
11. Prove that the sequence $\left({1}/{n}\right)$ is a Cauchy sequence.

12. If $f : A \to \mathbb{R}$ is a Lipschitz function, prove that f is uniformly continuous on A.

13. Prove that the sequence $\left(\frac{\sin n}{n}\right)$ converges to zero.

14. If $X = (x_n)$ and $Y = (y_n)$ are convergent sequences of real numbers and if $x_n \le y_n$ for all $n \in \mathbb{N}$, prove that $\lim(x_n) \le \lim(y_n)$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. State and prove Bernoulli's inequality.

16. If x and y are real numbers with x < y, show that there exists a rational number $r \in Q$ such that $x < r < y$.

17. If S is a nonempty subset of $\mathbb{R}$ that is bounded above and $a \in \mathbb{R}$, show that $\sup(a + S) = a + \sup S$, where $a + S = \{a + x : x \in S\}$.

18. Using the definition of the limit of a sequence, prove that $\lim\left(n/(n^2+1)\right) = 0$.

19. Prove that the unit interval [0, 1] is not countable.

20. If c > 0, prove that $\lim\left(c^{1/n}\right) = 1$.

21. If $X = (x_n)$, $Y = (y_n)$ and $Z = (z_n)$ are sequences of real numbers such that $x_n \le y_n \le z_n$ for all $n \in \mathbb{N}$ and that $\lim(x_n) = \lim(z_n)$, prove that $Y = (y_n)$ is convergent and $\lim(x_n) = \lim(y_n) = \lim(z_n)$.

22. If (x_n) is a sequence of positive real numbers such that $L = \lim\left(\frac{x_{n+1}}{x_n}\right)$ exists and L < 1, then prove that the sequence (x_n) converges to zero.

23. Suppose $a \in \mathbb{R}$ and a > 0. Construct a sequence (s_n) of real numbers that converges to $\sqrt{a}$.

24. If $X = (x_n)$ is a sequence of real numbers, prove that there is a subsequence of X that is monotone.

25. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

26. If a series $\sum x_n$ is convergent, prove that any series obtained from it by grouping terms is also convergent and to the same value.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Given that S is a subset of $\mathbb{R}$ that contains at least two points and has the property : if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$. Then prove that S is an interval.

28. Let $X = (x_n)$ be a sequence of real numbers. Then the following are equivalent.
    i) The sequence $X = (x_n)$ does not converge to $x \in \mathbb{R}$.
    ii) There exists an $\varepsilon_0 > 0$ such that for any $k \in \mathbb{N}$, there exists $n_k \in \mathbb{N}$ such that $n_k \ge k$ and $|x_{n_k} - x| \ge \varepsilon_0$.
    iii) There exists an $\varepsilon_0 > 0$ and a subsequence $X' = (x_{n_k})$ of X such that $|x_{n_k} - x| \ge \varepsilon_0$ for all $k \in \mathbb{N}$.

29. If I = [a, b] is a closed bounded interval and $f : I \to \mathbb{R}$ is continuous on I, prove that f has an absolute maximum and absolute minimum on I.

30. Let I be a closed bounded interval and $f : I \to R$ is continuous on I. Then prove that f is uniformly continuous on I.

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