Reg. No. : ....

Name : ....

K19U 3015

III Semester B.Sc. Hon's(Mathematics) Degree (Reg./Supple./Improv.)

Examination, November- 2019

(2016 Admission Onwards)

BHM 301 : REAL ANALYSIS

Time : 3 Hours | Max. Marks : 60

SECTION - A

Answer any **4** questions out of **5** questions. Each question carries **one** mark.

(4×1=4)

1. If z and a are elements in $\mathbb{R}$ with z+a=a, then z = ...
2. State the completeness property of $\mathbb{R}$, where $\mathbb{R}$ is the set of all numbers.
3. State whether the sequence $(n^2/n+1)$ is convergent or divergent.
4. Examine whether the following statement is true or false : If X is a sequence of nonzero real numbers such that X converges to $x \neq 0$ and Y is a sequence of real numbers such that XY converges, then Y is convergent.
5. Determine the points of continuity of the function f(x)=[x], where [x] is the greatest integer function.

SECTION- B

Answer any **6** questions out of **9** questions. Each question carries **2** marks

(6×2=12)

6. If $n \in \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, prove that n>0.
7. If a and b are real numbers such that a.b>0, then prove that either a>0 and b>0 or a<0 and b<0.

P.T.O.

8. Prove that $|a+b| \leq |a|+|b|$, where $a,b \in \mathbb{R}$.

9. If x and y are real numbers with x<y show that there exists an irrational number z such that x<z<y.

10. If y>0, show that there exists $n_y \in \mathbb{N}$ -such that $n_y - 1 \leq y \leq n_y$

11. Prove that every convergent sequence of real numbers is a Cauchy sequence.

12. If f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and if they are both bounded on A, prove that their product fg is uniformly continuous on A.

13. If $X=(x_n)$ and $y=(y_n)$ are convergent sequence of real numbers and if $x_n \leq y_n$ for all $n \in \mathbb{N}$, prove that $\lim(x_n) \leq \lim(y_n)$.

14. If $X=(x_n)$ is a convergent sequence and if $a \leq x_n \leq b$ for all $n \in \mathbb{N}$, prove that $a \leq \lim(x_n) \leq b$.

SECTION- C

Answer any **8** questions out of **12** questions. Each question carries **4** marks. **(8×4=32)**

15. Prove that there does not exist a rational number r such that $r^2=2$.

16. The function f is defined by $f(x) = \dfrac{2x^2+3x+1}{2x-1}$, for $2 \leq x \leq 3$. Find a constant M such that $|f(x)| \leq M$ for all x satisfying $2 \leq x \leq 3$.

17. If $S = \left\{\dfrac{1}{n} : n \in \mathbb{N}\right\}$, show that supS=1 and infS=0.

18. State and prove the 'Nested Interval Property'.

19. Prove that the set $\mathbb{R}$ of real numbers is not countable.

20. If $X = (x_n : n \in \mathbb{N})$ is a sequence of real numbers and $m \in \mathbb{N}$, prove that the m-tail $X_m = (x_{m+n} : n \in \mathbb{N})$ of X converges if and only if X converges.

21. Show that the sequence $\left(\frac{1}{1+na}\right)$ converges to zero, where a>0.

22. Given a sequence $X=(x_n)$ of real numbers such that it converges to x and $x_n \geq 0$ for all n. Show that the sequence $\left(\sqrt{x_n}\right)$ of positive square roots converges and $\lim\left(\sqrt{x_n}\right) = \sqrt{x}$.

23. The sequence $Z=(z_n)$ of real numbers is defined by $z_1=1$ and $z_{n+1} = \sqrt{2z_n}, n \in \mathbb{N}$ show that $\lim(z_n) = 2$.

24. If $X=(x_n)$ is a sequence of real numbers, then prove that there is a subsequence of X that is monotone.

25. Prove that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

26. If a series in $\mathbb{R}$ is absolutely convergent, prove that it is convergent.

SECTION- D

Answer any **2** questions out of **4** Questions. Each question carries **6** marks. **(2×6=12)**

27. State and prove the Monotone Convergence Theorem.

28. If (x_n) is a bounded sequence of real numbers, prove that the following statements for a real number x^* are equivalent

    a) $x^* = \limsup(x_n)$

    b) If $\varepsilon > 0$; there are atmost a finite number of $n \in \mathbb{N}$ such that $x^* + \varepsilon < x_n$, but an infinite number of $n \in \mathbb{N}$ such that $x^* - \varepsilon < x_n$.

    c) If $u_m = \sup\{x_n : n \geq m\}$, then $x^* = \inf\{u_m : m \in \mathbb{N}\} = \lim(u_m)$.

    d) If S is the set of subsequential limits of (x_n), then $x^* = \sup S$.

29. Define a contractive sequence and prove that every contractive sequence is convergent.

30. State and prove the location of roots theorem.
