Reg. No. : ....

Name : ....

K18U 2253

III Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2018
(2016 Admission Onwards)
BHM 301 : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. State Bernoulli's inequality.

2. Find all the real numbers x that satisfy the inequality $\frac{1}{x} < x^2$.

3. If $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$, find sup S.

4. Give an example of an unbounded sequence that has a convergent subsequence.

5. State whether the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ is convergent or divergent.

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks. **(6×2=12)**

6. If a is a real number, prove that $-|a| \le a \le |a|$.

7. If A and B are nonempty subsets of $\mathbb{R}$ that satisfy the property : $a \le b$ for all $a \in A$ and $b \in B$, prove that $\sup A \le \sup B$.

8. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, prove that $\inf S = 0$.

9. Given x and y are real numbers with $x < y$, then prove that there exists an irrational-number z such that $x < z < y$.

P.T.O.

10. Show that $\lim \left(\frac{1}{n} - \frac{1}{n+1}\right) = 0$.

11. If $X = (x_n)$ is a convergent sequence of real-numbers and if $x_n \geq 0$ for all $n \in \mathbb{N}$, show that $x = \lim (x_n) \geq 0$.

12. Prove that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence.

13. If a series in $\mathbb{R}$ is absolutely convergent, prove that it is convergent.

14. If the series $\sum x_n$ is convergent, prove that any series obtained from it by grouping the terms is also convergent.

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks. **(8×4=32)**

15. Determine the set $B = \{x \in \mathbb{R} : x^2 + x > 2\}$.

16. If $a, b \in \mathbb{R}$, prove that $\left||a| - |b|\right| \leq |a - b|$.

17. State and prove the 'Nested Interval Property'.

18. Using the definition of limit, show that $\lim \left(\frac{n}{n^2+1}\right) = 0$.

19. Prove that the set $\mathbb{R}$ of real numbers is not countable.

20. If $X = (x_n)$ is a sequence of real numbers, prove that there is a subsequence of X that is monotone.

21. Prove that every convergent sequence is a Cauchy sequence.

22. Prove that the harmonic series $\sum_{n=1}^{\infty} 1/n$ diverges.

23. If $X = (x_n)$ is a contractive sequence with constant c, $0 < c < 1$ and if $x^* = \lim X$, prove that $|x^* - x| \leq \frac{c}{1-c}|x_n - x_{n-1}|$.

24. State and prove the Bolzano's intermediate value theorem.

25. If $f : A \to \mathbb{R}$, $A \subseteq \mathbb{R}$, is uniformly continuous on A and if (x_n) is a Cauchy sequence in A, show that $(f(x_n))$ is a Cauchy sequence in $\mathbb{R}$.

26. If $X = (x_n)$ is a convergent monotone sequence and the series $\sum y_n$ is convergent, prove that the series $\sum x_n y_n$ is also convergent.

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks. **(2×6=12)**

27. If $x \in \mathbb{R}$, prove that there exists $n_x \in \mathbb{N}$ such that $x \leq n_x$.

28. Prove that there exists a positive real number x such that $x^2 = 2$.

29. State and prove the Cauchy convergence criterion for a sequence of real numbers.

30. Let $I = [a, b]$ and let $f : I \to \mathbb{R}$ be continuous on I. If $f(a) < 0 < f(b)$ or if $f(a) > 0 > f(b)$, prove that there exists a number $c \in (a, b)$ such that $f(c) = 0$.