Reg. No. : ....

Name : ....

K15U 0620

III Semester B.Sc. Hon's (Maths) Degree (Reg./Supple./Improve.) Examination, November 2015
BHM 301 : REAL ANALYSIS – I

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Determine the set A of all real numbers such that $2x + 3 \le 6$.
2. Show that $|x - a| < \varepsilon$ if and only if $a - \varepsilon < x < a + \varepsilon$.
3. Find sup $\left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$.
4. Show that the sequence $\left(\frac{1}{n^2 + 1}\right)$ converger to zero.
5. If $X = (x_n)$ is a sequence of real numbers, define the m-tail of X.
6. Define the subsequence of a sequence of real numbers.
7. State the Cauchy criterion for the convergence of a series.
8. State the alternating series test.
9. Show that the series $\sum_{n=0}^{\infty}(-1)^n$ generated by the sequence $\left((-1)^n\right)$ is divergent.
10. Give an example of a uniformly continuous function which is not a Lipschitz function.

P.T.O.

Answer **any ten** short answer questions **out** of **14**. **(10×3=30)**

11. If $x > -1$, show that $(1+x)^n \geq 1+nx$ for all $n \in \mathbb{N}$.

12. If a, $b \in \mathbb{R}$, prove that $\| a | - | b \| \leq | a - b |$.

13. If x and y are real numbers with x < y, show that there exists a rational number r such that x < r < y.

14. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, show that inf S = 0.

15. If $X = (x_n : n \in \mathbb{N})$ is a sequence of real numbers and $m \in \mathbb{N}$, prove that the m -tail $X_m = (x_{m+n} : n \in \mathbb{N})$ of X converges if and only if X converges.

16. If $X = (x_n)$ is a convergent sequence and if $a \leq x_n \leq b$ for all $n \in \mathbb{N}$, show that $a \leq \lim (x_n) \leq b$.

17. Show that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence.

18. If $X = (x_n)$ converges to x, show that the sequence of absolute values converges to $|x|$.

19. Show that the geometric series $\sum_{n=0}^{\infty} r^n$ converges for $| r | < |$.

20. If (x_n) is a sequence of non-negative real-numbers, show that the series $\sum x_n$ converges if and only if the sequence S = (sk) of partial sums is bounded.

21. If $X = (x_n)$ is a sequence in $\mathbb{R}$ such that $|x_n|^{1/n} \leq r$, for $n \geq K, K \in \mathbb{N}$ and r > 1, Show that the series $\sum x_n$ is absolutely convergent.

22. If I is an interval, $f : I \rightarrow \mathbb{R}$ is continuous on I and $f(a) < k < f(b)$, for $k \in \mathbb{R}$, a, $b \in I$, show that there exists a point $c \in I$ between a and b such that f (c) = k.

23. If I is an interval and $f : I \rightarrow \mathbb{R}$ is continuous, show that f (I) is an interval.

24. If $f : A \rightarrow \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in R, show that $(f(x_n))$ is a Cauchy sequence in $\mathbb{R}$.

Answer **any six** short essay questions out of **nine**. **(6×5=30)**

25. If $f(x) = \frac{2x^2+3x+1}{2x-1}, 2 \leq x \leq 3$, find a constant M such that $| f(x) | \leq M$.

26. If S is a non-empty subset of $\mathbb{R}$ that is bounded above ; prove that sup (a + S) = a + sup S, where $a \in \mathbb{R}$ and $a + S = \{a + s : s \in S\}$.

27. If S is a subset of $\mathbb{R}$ that contain atleast two points and has the property $[x, y] \subseteq S$ – whenever x, $y \in S$, x < y, prove that S is an interval.

28. Show that the set $\mathbb{R}$ of real numbers is not countable.

29. If $c > \theta$, show that $\lim \left(C^{1/n}\right) = 1$.

30. Prove that every contractive sequence in covergent.

31. If $X = (x_n)$ is a decreasing sequence with $\lim (x_n) = 0$ and if the partial sums (Sn) of $\sum y_n$ are bounded, prove that the series $\sum x_n y_n$ is convergent.

32. Show that $f(x) = 1/x$ for $x \in A = \{x \in \mathbb{R} : x > 0\}$ is continuous but not uniformly continuous on A.

33. If I is a closed bounded interval and $f : I \rightarrow \mathbb{R}$ is continuous on I, show that for every $\varepsilon > 0$, there exists a continuous piece wise linear formation $g : I \rightarrow \mathbb{R}$ such that $|f(x) - g_\varepsilon(x)| < \varepsilon$ for all $x \in I$.

Answer **any one** essay question out of **two**. **(1×10=10)**

34. Prove that there exists a real number x such that $x^2 = 2$.

35. State and prove the monotone subsequence theorem for a sequence of real numbers. Using this prove the Bolzano-Weier strass theorem.