32. Determine the eigen spaces corresponding the eigen values of the operator. $T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{pmatrix}$ on $\mathbb{R}^3$.

33. In $C([0, 1])$ with inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$. Compute $\langle f, g \rangle$, $||f||$ and $||f+g||$, where $f(t) = t^0$ and $g(t) = e^t$. Then verify Cauchy's inequality and triangular inequality. **(5×6=30)**

Answer **any one** question from the questions **34** and **35**. This question carries **10** marks.

34. Let V and W be vector spaces and let $T : V \to W$ be linear. If V is finite dimensional, then prove that R(T) is finite dimensional and nullity (T) + rank(T) = dim V.

35 a) State and prove Gram-Schmidth orthogonalization process.

b) Use Gram-Schmidth orthogonalization process obtain an orthonormal basis corresponding to the set {(1, 1, 0), (2, 0, 1), (2, 2, 1)}. **(10×1=10)**

K17U 2669

Reg. No. : ....

Name : ....

III Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2017
(2013-15 Admissions)
BHM 304 : LINEAR ALGEBRA – I

Time : 3 Hours Max. Marks : 80

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. How many elements are there in the vector space $M_{m\times n}(\mathbb{Z}_2)$?

2. Is the set $W = \{f \in P(F) \mid f = 0 \text{ or } f \text{ has degree } n\}$ a subspace of P(F) if $n \geq 1$? Justify your answer.

3. Determine whether the vector (3, 4, 1) can be written as a linear combination of (1, –2, 1) and (–2, –1, 1).

4. What is the dimension of the vector space $M_{m\times n}(\mathbb{R})$?

5. Check whether $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$ a linear transformation or not.

6. Find the matrix with respect to the standard basis for the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a, - 4a_2)$.

7. Does there exists an invertible linear transformation from $\mathbb{R}^3$ onto $\mathbb{R}^2$? Justify your answer.

8. Find all the eigen values of the matrix $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.

9. Find the conjugate transpose of the matrix $A = \begin{bmatrix} i & 1+2i \\ 2 & 3+4i \end{bmatrix}$.

10. Define an orthonormal basis in an inner product space. **(1×10=10)**

Answer **any 10** questions from among the questions **11** to **24**. These questions carries **3** marks **each**.

11. Let V denote the set of all ordered pairs of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and C an element of F, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2b_2)$ and $c(a_1, a_2) = (ca_1, a_2)$. Is V a vector space under these operations ? Justify your answer.

12. Is the set of all differentiable real valued functions defined on $\mathbb{R}$ a subspace of $C(\mathbb{R})$? Justify.

13. Find span $\left(\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\right)$ in $M_{2\times2}(F)$.

14. Show that a subset W of a vector space V is a subspace of V if and only if span (W) = W.

15. Show that the vectors (1, –3, 2), (4, 1, 0) and (0, 2, –1) is a basis of $\mathbb{R}^3$.

16. Do the polynomials $x^3 - 2x^2 + 1$, $4x^2 - x + 3$ and $3x - 2$ generate $P_3(\mathbb{R})$? Justify.

17. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear and that T(1, 0) = (1, 4) and T(1, 1) = (2, 5). What is T(2, 3) ?

18. Find the null space N(T) of the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$.

19. Find the matrix of the linear transformation $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ by $T(f) = f'$ with respect to the basis $\beta = \{1, x, x^2, x^3\}$. Also find the null space of T.

20. Find the characteristic polynomial of the linear transformation $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ by $T(f) = f'$.

21. Find the eigen values of the matrix $A = \begin{bmatrix}1 & 1\\4 & 1\end{bmatrix}$.

22. Show that the matrix $A = \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ in $M_{2\times2}(\mathbb{R})$ is diagonalizable.

23. Use Cayley-Hamiton theorem, find the square of the matrix $A = \begin{bmatrix}1 & 2\\-2 & 1\end{bmatrix}$.

24. Let V be an inner product space. Show that $\|x + y\| \le \|x\| + \|y\|$ for all $x, y \in V$.

(3×10=30)

Answer **any 6** questions from among the questions **25** to **33**. These questions carry **5** marks **each**.

25. Let W be a subspace of a finite-dimensional vector space V. Then prove that W is finite-dimensional and $\dim(W) \le \dim(V)$.

26. Let W_1 and W_2 be subspaces of V having dimension m and n respectively with $m \ge n$. Then
    a) Show that $\dim(W_1 \cap W_2) \le n$ and $\dim(W_1 + W_2) \le m + n$.
    b) Give examples of subspaces W_1 and W_2 of $\mathbb{R}^3$ for which $\dim(W_1 \cap W_2) = n$ and $\dim(W_1 + W_2) = m + n$.

27. Show that any maximal linear independent set in a finite-dimensional vector space is a basis.

28. Define $T : P_2(\mathbb{R}) \to M_{2\times2}(\mathbb{R})$ by $T(f) = \begin{pmatrix}f(1) - f(2) & 0\\0 & f(0)\end{pmatrix}$. Find a basis for R(T).

29. Prove that a linear transformation is one-to-one if and only if N(T) = {0}.

30. Show that F^2 is isomorphic to $P_1(F)$.

31. Show that non-zero eigen vectors corresponding to distinct eigen values of a operator is linearly independent.