Reg. No.: ....

Name: ....

K16U 2584

III Semester B.Sc. Hon's (Mathematics) Degree (Reg. Supple./Improv.) Examination, November 2016
BHM304 : LINEAR ALGEBRA – I

Time : 3 Hours   Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Define subspace of a vector space.

2. What do you mean by spanning set of a vector space ?

3. What do you mean by basis of a vector space ?

4. What is the dimension of the vector space of complex numbers over the field of real numbers ? Also give a basis this vector space.

5. If T is a linear transformation, then prove that T(0) = 0.

6. What do you mean by nullity of a linear transformation ?

7. What do you mean by similar matrices ?

8. Define eigen vectors of a linear transformation.

9. Define an inner product.

10. What do you mean by orthogonal set ?

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. Prove that the additive inverse of an element in a vector space is unique.

12. Prove that set of all symmetric matrices of order n is a subspace of the vector space of all square matrices of order n with entries from a field F.

13. Determine whether the set {(1, – 1, 2), (2, 0, 1), (–1, 2, –1)} is linearly dependent or not.

14. Let V be a vector space with dimension n. Prove that every linearly independent subset of V can be extended to a basis for V.

15. Prove that {(1, 0, 0, – 1), (0, 1, 0, –1), (0, 0, 1, –1), (0, 0, 0, 1} is a basis for R^4.

16. Define $T: R^2 \to R^3$ by T (x, y) = (x + y, 0, 2x – y). Check whether T is linear or not.

17. Let V, W and Z be vector spaces over the same field F and let $T: V \to W$ and $U: W \to Z$ be linear. Prove that $UT: V \to Z$ is linear.

18. Prove that $P_3(R)$, the set of all polynomials of degree less than or equal to 3 over R is isomorphic to $M_{2\times 2}(R)$.

19. Let $\beta = \{(1, 0), (0, 1)\}$ and $\beta' = \{(a_1, a_2), (b_1, b_2)\}$ be the ordered bases for R^2. Find the change of coordinate matrix that changes β' – coordinates into β – coordinates.

20. Let V be a finite dimensional vector space and let $x \in V$. If $\hat{x}(f) = 0$ for all $f \in V^*$, then prove that x = 0.

21. Find the eigen values of $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

22. Let T be a linear operator on P(R) defined by $T(f(x)) = f'(x)$. Find an ordered basis for the T-cyclic subspace generated by x^2.

23. State and prove Triangle Inequality in an inner product space.

24. Let T be a linear operator on an inner product space V and suppose that $\|T(x)\| = \|x\|$ for all x. Prove that T is one-to-one.

Answer **any 6** short answer questions out of 9 : **(6×5=30)**

25. Prove that intersection of two subspaces of a vector space is again a subspace. What about the union of two subspaces ? Justify your answer.

26. Let u and v be distinct vectors in a vector space V. Show that {u, v} is linearly dependent if and only if u or v is a multiple of other.

27. Let V be a vector space and S a subset that generates V. If β is a maximal linearly independent subset of S, then prove that β is a basis for V.

28. Let $T: P_3(R) \to P_2(R)$ be a linear transformation defined by $T(f(x)) = f'(x)$. Let β and γ be the standard ordered bases for $P_3(R)$ and $P_2(R)$, respectively. Compute $[T]_\beta^\gamma$.

29. Let V be a finite dimensional vector space and define $\psi: V - V^{**}$ by $\psi(x) = \hat{x}$. Show that ψ is an isomorphism.

30. Let T be a linear operator on a finite dimensional vector space V and let λ be an eigen value of T having multiplicity m. Prove that $1 \le \dim(E_\lambda) \le m$.

31. Verify Cayley-Hamilton theorem for $T: R^2 \to R^2$ defined by $T(a,b) = (a + 2b, -2a + b)$.

32. Using Gram-Schmidt process obtain an orthonormal basis for span {1. x, x^2} in $P_2(R)$, the set of all polynomials of degree less than or equal to 2 over R.

33. If W is any subspace of a vector space V, prove that dim (V) = dim(W) + $\dim(W^\perp)$.

Answer **any one** essay question out of 2 : **(1×10=10)**

34. State and prove dimension theorem.

35. Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{bmatrix}$. Find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.