K15U 0623

Reg. No. : ....

Name : ....

III Semester B.Sc. Hon's (Maths) Degree (Reg./Supple./Improve.) Examination, November 2015
BHM 304 : LINEAR ALGEBRA – I

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Give an example for a vector space.
2. What do you mean by generating set of a vector space ?
3. What do you mean by dimension of a vector space ?
4. What is the standard basis of P(F), the set of all polynomials with coefficients from F ?
5. Define a linear transformation.
6. What do you mean by rank of a linear transformation ?
7. Define isomorphism between two vector spaces.
8. Define Eigen values of a linear transformation.
9. Define a norm.
10. Give an example for an orthonormal set in R^3.

Answer **any 10** short answer questions **out** of **14**. **(10×3=30)**

11. State and prove cancellation law for vector addition.
12. Prove that set of all $n \times n$ matrices having trace equal to zero is a subspace of the $M_{n\times n}(F)$, the set of all square matrices of order n with entries from a field F.

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13. Prove that {(1, 0, 0, –1), (0, 1, 0, – 1), (0, 0, 1, –1), (0, 0, 0, 1)} is linearly independent in R^4.

14. Let V be a vector space with dimension n. Prove that any lineraly independent subset of V that contains exactly n vectors is a basis for V.

15. Prove that $\left\{\begin{bmatrix}1&1\\1&0\end{bmatrix},\begin{bmatrix}1&1\\0&1\end{bmatrix},\begin{bmatrix}1&0\\1&1\end{bmatrix},\begin{bmatrix}0&1\\1&1\end{bmatrix}\right\}$ is a basis for $M_{2x2}(R)$.

16. Define $T: R^2 \to R^2$ by T(x, y) = (x+1,y). Check whether T is linear or not ?

17. Let A be ab $m \times n$ matrix and B be an $n \times p$ matrix. For each $j(1 \leq j \leq p)$ let u_j and v_j denote the jth columns of AB and BA respectively. Prove that $u_j = Av_j$.

18. Prove that F^2 is isomorphic to $P_1(F)$, the set of all first degree polynomials over F.

19. Let β = {(2, 5), (–1, –3)} and β' = {(1, 0), (0, 1)} be the ordered bases for R^2. Find the change of coordinate matrix that changes β'-coordinates into β-coordinates.

20. Let β= {(1, 0, 1), (1, 2, 1), (0, 0, 1)} be an ordered basis for R^3. Find explicit formulas for vectors of the dual basis β^* for V^*.

21. Let $A \in M_{n \times n}(F)$. Prove that a scalar λ is an Eigen value of A if and only if $\det(A - \lambda I_n) = 0$.

22. Let T be a linear operator on R^3 defined by T(a, b, c) = (–b + c, a + c, 3c). Find an ordered basis for the T-cyclic subspace generated by the vector e_1 = (1, 0, 0).

23. State and prove Cauchy-Schwarz Inequality in an inner product space.

24. What do you mean by orthogonal complement of a subset of an inner product space ? What is the orthogonal complement of S = {(0, 0, 1)} in R^3 ?

Answer **any 6** short answer questions **out** of **9** : **(6×5=30)**

25. Let V be a vector space and W a subset of V. Prove that W is a subspace of V if and only if a) $0 \in W$, b) $x + y \in W$ whenever $x \in W$ and $y \in W$ and c) $kx \in W$ whenever $k \in F$ and $x \in W$.

26. Let u and v be distinct vectors in a vector space V. Show that {u, v} is linearly dependent if and only if u or v is a multiple of other.

27. Let W be a subspace of a finite dimensional vector space V. Then prove that W is finite dimensional and dim(W) $\leq$ dim (V).

28. Let $T : R^2 \to R^3$ be a linear transformation defined by T(x, y) = (x + 3y, 0, 2x – 4y). Let β and γ be the standard ordered bases for R^2 and R^3, respectively. Compute $[T]_\beta^\gamma$.

29. Let V be a finite dimensional vector space and define $\psi : V \to V^{**}$ by $\psi(x) = \hat{x}$. Show that ψ is an isomorphism.

30. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix}1&1\\4&1\end{bmatrix}$.

31. State and prove Cayley-Hamilton theorem for linear operators.

32. Using Gram-Schmidt process obtain an orthonormal basis for span {(1, 0, 1, 0), (1, 1, 1, 1), (0, 1, 2, 1)} in R^4.

33. Suppose that $S = \{v_1, v_2, ..., v_k\}$ is an orthonormal set in an n-dimensional inner product space V. Prove that S can be extended to an orthonormal basis for V.

Answer **any one** essay questions **out** of **2** : **(1×10=10)**

34. State and prove dimension theorem.

35. Let T be a linear operator on $V = P_2(R)$ defined by $T(f(x)) = f(1) + f'(0)\,x + (f'(0) + f''(0))x^2$. Test T for diagonalizability and if T is diagonalizable, find a basis β for V such that $[T]_\beta$ is a diagonal matrix.
