Reg. No. : ....

Name : ....

M 7965

III Semester B.Sc. Hon's (Mathematics) Degree (Regular) Examination, November 2014
BHM 303 : DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Verify whether $y = x^2$ is a solution of the differential equation $xy' = 2y$.
2. Examine whether the equation $2xy\,dx + x^2\,dy = 0$ is exact.
3. State the existence theorem for the solution of the initial value problem $y' = f(x, y),\ y(x_0) = y_0$.
4. Solve $y'' - y = 0$.
5. Give the formula for the particular solution y_p of the differential equation $y'' + p(x)y' + q(x)y = r(x)$, with arbitrary variable functions that are continuous on some interval I.
6. Write down the solution of the exact differential equation $P(x, y)\,dx + Q(x, y)\,dy = 0$.
7. Find an integrating factor of the differentiate equation $2\sin(y^2)\,dx + xy\cos(y^2) = 0$.
8. Write down the solution of the linear differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x.
9. Give the general solution if the characteristic equation of the differential equation $y'' + ay' + by = 0$ has two distinct roots.
10. Obtain the auxiliary equation of the Euler-Cauchy equation $x^2y'' + axy' + by = 0$.

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Answer **any ten** short answer questions out of 14 : **(10×3=30)**

11. Solve the differential equation $ayy' + 4x = 0$.

12. Solve : $y' - y = 4$.

13. Find the orthogonal trajectories of $y = cx^2$.

14. Show that e^x is an integrating factor of the equation $\sin y\, dx + \cos y\, dy = 0$ and solve it.

15. Show that any linear combination of two solutions of the homogeneous linear differential equation $y'' + p(x)y' + q(x)y = 0$ on an open interval I is again a solution.

16. Solve the initial value problem $y'' + 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

17. Find a general solution of the equation $x^2 y'' - 20y = 0$.

18. If p(x) and q(x) are continuous on an open interval I, show that the equation $y'' + p(x)y' + q(x)y = 0$ has a general solution on I.

19. Determine the type and stability of the critical point of the system $y_1^1 = y_1 + 2y_2$, $y_2^1 = 2y_1 + y_2$.

20. Explain Picard's method of successive approximation of solution of $y' = f(x, y)$, $y(x_0) = y_0$.

21. Obtain the second order Runje-Kutta formula for solving the equation $y' = f(x, y)$, $y(x_0) = y_0$.

22. Given the differential equation $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0$. Find y(0.2) using Runje-Kutta fourth order formula (h = 0.2).

23. State Milne's predictor-corrector formula for the solution of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$.

24. Solve the equation $y'' - y = 0$ by converting it to a system of two first order equations.

Answer **any six** short essay questions out of nine : **(6×5=30)**

25. Solve : $2xyy' = y^2 - x^2$.

26. Solve : $y' + \frac{1}{x}y = 3x^2y^2$.

27. Find the approximate solutions $y^{(1)}$ and $y^{(2)}$ to the initial value problem $y^1 = x + y^2$ by Picard's iteration subject to the condition y(0) = 1.

28. Solve : $4y'' - 4y' - 3y = 0$, $y(-2) = e$, $y'(-2) = -e/2$.

29. Verify $y_p = 2x$ is a solution of the equation $y'' + y = 2x$ and solve the initial value problem $y'' + y = 2x$, $y(0) = -1$, $y'(0) = 8$.

30. Solve : $(D^2 + 1)y = e^{-x}$, $y(0) = -1$, $y'(0) = -1$.

31. Solve the differential equation $y'' + y = \sec x$ if $y_1 = \cos x$ and $y_2 = \sin x$ are a basis of solutions of the homogeneous equation $y'' + y = 0$.

32. Find a general solution of the system of equations $y_1' = -3y_1 + y_2$, $y_2' = y_1 - 3y_2$.

33. If $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$, obtain the Taylor series for y(x) and compute y(0.1) correct to three decimal places.

Answer **any one** essay equation out of two : **(10×1=10)**

34. If y_1 and y_2 are the solutions of the homogeneous linear equation $y'' + p(x)y' + q(x)y = 0$, show that $y_2 = y_1 \int U dx$, where $U = \frac{1}{y_1^2} e^{-\int P dx}$.

35. Given the differential equation $y' = x^2 + y$ with $y(0) = 1$, compute y when x = 0.1 and when x = 0.15 correct to three decimal places by using Euler's modified method (h = 0.05).

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