31. A random sample of n observations $Y_1, \ldots, Y_n$ is selected from a population with Gamma probability distribution with parameters α and β. Find the method of moments estimators for the unknown parameters α and β.

32. Briefly describe the steps in test of significance.

33. A psychological study was conducted to compare the reaction times of men and women to a stimulus. Independent random samples of 50 men and 50 women were employed in the experiment. The results are as follows :

Men	$n_1 = 50$	$\bar{y}_1 = 3.6$ seconds	$S_1^2 = 0.18$
Women	$n_2 = 50$	$\bar{y}_2 = 3.8$ seconds	$S_2^2 = 0.14$

Do the data presents sufficient evidence to suggest a difference between true mean reaction times for men and women ? Use $\alpha = 0.05$. Find the p-value.

Answer **any one** essay questions out of **2**. **(1×10=10)**

34. Suppose that $Y_1, \ldots, Y_n$ constitute a random sample from a normal distribution $N(\mu, \sigma)$. We wish to test $H_0 : \mu = \mu_0$ against the alternative $H_0 : \mu > \mu_a$, for a specified constant μ_0. Find the uniformly most powerful test with significance level α.

35. Find an unbiased estimate of population variance in case of a normal distribution. A random sample of 12 values gave the unbiased estimate of population variance equal to 10.62. Calculate 95% confidence interval for the population variance.

Reg. No. : ....

Name : ....

K15U 0624

III Semester B.Sc. Hon's (Maths) Degree (Reg./Supple./Improve.) Examination, November 2015

BHM 305 : ADVANCED STATISTICS – I

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. When do you say that a point estimator is unbiased ?
2. Explain the error of an estimation.
3. Define Confidence coefficient.
4. Define the mean square error of a point estimator.
5. If μ is a statistic based ion the random sample $Y_1, \ldots, Y_n$, give a necessary sufficient condition for μ to be a sufficient estimation of parameter θ.
6. Give a consistent estimator of the population variance in case of normal distribution.
7. Explain the term rejection region.
8. Give an example of test statistic following F-distribution.
9. Give the test statistic for testing whether a normally distributed population has a given variance when the sample variance is given.
10. Define the power of a test.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. A random sample of size 17 from a normal population is found to have mean 4.7 and variance 5.76. Find a 90% confidence interval for the mean of the population.

12. Find the maximum likelihood estimate for the mean in case of a normal population.

13. Let $Y_1, \ldots, Y_n$ denote a random sample from a distribution with mean μ and variance σ^2. Show that $\bar{Y}_n = \frac{\sum_{i=1}^{n} Y_i}{n}$ is a consistent estimator of μ.

14. Let $Y_1, \ldots, Y_n$ denote a random sample from uniform distribution on the interval $(\theta, \theta + 1)$. Let $\theta_1 = \overline{Y} - \frac{1}{2}$ where $\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$. Show that θ_1 is an unbiased estimate of θ.

15. Suppose $Y_1, \ldots, Y_n$ constitute a random sample from a Poisson distribution with mean λ. Find the method of moments estimator of λ.

16. Suppose $Y_1, \ldots, Y_n$ constitute a random sample from distribution where $P(Y_i = 1) = p$ and $P(Y_i = 0) = 1 - p$ with p unknown. Use the factorization criteria to find a sufficient statistic that best summarizes the data.

17. The standard deviations calculated from two random samples of sizes 9 and 13 are 2.1 and 1.8 respectively. Can the samples be regarded as drawn from normal populations with same SD ?

18. From a population with unknown SD a sample of size n was taken and its mean and SD were found to be 195 and 50. If the hypothesis that mean of the population is 200 rejected at 5% level of significance what can be said about sample size.

19. A study by children's hospital indicate that 67% of adults and 15% of children are overweight. Thirteen children in a random sample of size 100 were found to be overweight. Is there sufficient evidence to indicate that the percentage reported by children's hospital is too high. Test at the $\alpha = 0.05$ level of significance.

20. A manufacturer of gun powder has developed a new powder which was tested in eight shells. The resulting muzzle velocities had mean 2959 ft/s and SD = 39.1. The manufacturer claims that the new gun-powder produces an average velocity of not less than 3000 feet per second. Do the sample data provide sufficient evidence to contradict the manufacturer's claim at 0.025 level of significance.

21. The fraction of defective items in a large lot is P. To test the null hypothesis $H_0 : P = 0.2$ one considers the number f of defectives in a sample of 8 items and accepts the hypothesis if $f \leq 6$ and rejects the hypothesis otherwise. What is the probability of type-II error corresponding to P = 0.1 ?

22. In a city the milk consumption of families, X is assumed to follow the distribution

$$f(x, \theta) = \begin{cases} \left(\frac{1}{\theta}\right) e^{\frac{-x}{\theta}} \\ 0 \text{ elsewhere} \end{cases}$$

Where $\theta > 0$. The hypothesis $H_0 : \theta = 5$ is rejected in favour of $H_a : \theta = 10$ if a family selected at random consumes 15 units or more. Find the value of α.

23. The standard deviation of sample size 15 from a normal population was found to be 7. Examine whether the hypothesis that SD is more than 7.6 is acceptable $(\alpha = 0.05)$.

24. Distinguish between statistic and parameter giving examples.

Answer **any 6** short answer questions out of **9**. **(6×5=30)**

25. Let $Y_1, \ldots, Y_n$ constitute a random sample from a normal distribution with unknown mean μ and variance σ^2. Find minimum variance unbiased estimates for μ and σ^2.

26. On the basis of a random sample find the maximum likelihood estimator of the parameter of a Poisson distribution.

27. A random sample of 10 students of class II was selected from schools in a certain region. Their weights recorded are as follows :

38, 46, 45, 40, 35, 39, 44, 45, 33, 37

Find 95% confidence interval in which the mean weight of all such students in the region is expected to lie.

28. In a large city A, 20 percent of a random sample of 900 school children had defective eye sight. In another large city B 5 percent of a random sample of 1600 children had the same defect. Is this difference between the two proportions significant ?

29. Show that the mean and standard error of sample mean $(\overline{x})$ from simple samples of size n are given by $E(\overline{x}) = \mu$ and $SE(\overline{x}) = \frac{\sigma}{\sqrt{n}}$.

30. Find the Maximum likelihood estimates of a and b if the pdf of the population is given by $f(x) = \frac{1}{b-a}$, $a \leq x \leq b$.