Reg. No. : ....

Name : ....

K21U 0235

III Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2020
(2016 Admission Onwards)
BHM305 : ADVANCED LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

(Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.) **(4×1=4)**

1. Define Hyperspace.
2. Define characteristic value of a linear operator.
3. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$.
4. Define an inner product and an inner product space.
5. Prove that $\alpha/\beta = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$, where $\alpha = (x_1, y_1)$ and $\beta = (x_2, y_2)$ is an inner product on R_2.

SECTION – B

(Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.) **(6×2=12)**

6. Let V be a finite dimensional vector space over the field F. Prove that each basis for V^* is the dual of some basis for V.
7. Let A be any $m \times n$ matrix over the field F. Prove that the row rank of A is equal to the column rank of A.
8. Prove that the similar matrices have same characteristic polynomial.
9. Let V be a finite dimensional vector space. What is the minimal polynomial for the identity operator on V ? Why ?
10. Let W is an invariant subspace for T, then prove that W is invariant under every polynomial in T.

11. Prove that an orthogonal set of non-zero vectors is linearly independent.
12. Prove that every finite dimensional inner product space has an orthonormal basis.
13. Let V be an inner product space and T a self adjoint linear operator on V. Then prove that each characteristic value of T is real, and characteristic vectors of T associated with distinct characteristic values are orthogonal.
14. Let V be a finite dimensional inner product space, and let T be a linear operator on V. Suppose W is a subspace of V which is invariant under T. Then prove that the orthogonal complement of W is invariant under T^*.

SECTION – C

(Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.) **(8×4=32)**

15. If f is a non zero linear functional on the vector space V. Then prove that the null space of f is a hyperspace in V. Conversely, every hyperspace in V is the null space of a non zero linear functional on V.
16. Let $A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$ Is A similar over the field R to a diagonal matrix ?
17. Let T be a linear operator on an n-dimensional vector space V. Show that the characteristic and minimal polynomial for T have the same roots except for multiplicities.
18. Find the minimal polynomial for the matrix.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix}.$$

19. Prove that every matrix A such that $A^2 = A$ is similar to a diagonal matrix.
20. State and prove Cauchy-Schwartz inequality.
21. Describe the Gram-Schmidt orthogonalization process.
22. Consider vectors (3, 0, 4), (–1, 0, 7), (2, 9, 11) in R^3. Find an orthonormal basis for R^3.

23. Let W be a subspace of an inner product space V and Let β be a vector in V. Then prove that the vector α in W is a best approximation to β by vectors in W if and only if $\beta - \alpha$ is orthogonal to every vector in W.
24. State and prove Bessel's inequality.
25. Let V be a finite dimensional inner product space, and if is a linear functional on V. Then prove that there exists a unique vector β in V such that $f(\alpha) = (\alpha/\beta)$ for all α in V.
26. For every linear operator T on a finite dimensional inner product space V, prove that the exists a unique linear operator T^* on V such that $(T\alpha/\beta) = (\alpha/T^*\beta)$ for all α, β in V.

SECTION – D

(Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.) **(2×6=12)**

27. State and prove Cayley-Hamilton Theorem.
28. State and prove Primary Decomposition Theorem.
29. Prove that on a finite dimensional inner product space of positive dimension, every self adjoint operator has a non-zero characteristic vector.
30. Let V be a finite dimensional inner product space and T be a self-adjoint linear operator on V. Then prove that there is an orthonormal basis for V, each vector of which is a characteristic vector for T.

———————