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29. Let Wbe a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Prove the following.

(4)

- a) I-E is the orthogonal projection of V on W^{\perp}
- b) I-E is an idempotent linear transformation of V onto w^{\perp} with null space W.
- 30. Prove that for every in vertible complex nxn matrix B there exists a unique lower triangular matrix M with positive entries on the main diagonal such that MB is unitary.



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Name :

Reg. No.:....

III Semester B.Sc.Hon's (Mathematics) Degree (Reg./Supple./ Improv.)
Examination, November 2019

(2016 Admission onwards)

BHM 305: ADVANCED LINEAR ALGEBRA

Time: 3 Hours

Max. Marks: 60

SECTION - A

Answer any 4 questions out of 5. Each question carries One mark. (4×1=4)

- Define nilpotent operator.
- 2. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by T(x,y) = (x+y,0). Find null space of T.
- 3. Define projection of a vector space. Give an example.
- 4. Is it true that "every matrix A such that A2=A is diagonalizable."
- 5. Give an example of an orthogonal matrix which is not unitary.

SECTION - B

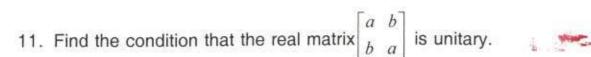
Answer any 6 out of 9. Each question carries 2 marks.

 $(6 \times 2 = 12)$

- 6. Define self adjoint operator. Give an example.
- 7. If S is any subset of a Finite dimensional vector space V, then prove that $(S^0)^0$ is the subspace spanned by S.
- 8. Let *V* and *W* be vector spaces over the field *F* and let *T* be a linear transformation from *V* into *W*. Prove that the null space of T^t is the annihilator of the range of *T*.
- 9. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find the minimal polynomial of the operator corresponding to the matrix A.
- 10. Let V be a vector space and (I) an inner product on V. Show that $(0 | \beta) = 0$ for all β in V.

P.T.O.





12. Let V be a finite - dimensional innerproduct space, and T a linear operation on V. Suppose W is a subspace of V which is invariant under T. Prove that the orthogonal compliment of W is invariant under T*

(2)

- Let V be a finite dimensional inner product space. If T is a linear operator on V, show that (T*)*=T.
- 14. Differentiate between orthonormal set and orthogonal basis with the help of suitable examples.

SECTION - C

Answer any 8 questions out of 12. Each question carries 4 marks.
(8×4=32)

- 15. If V is a finite dimensional vector space and $\alpha(\neq 0) \in V$, then prove that there exists a linear functional f such that $f(\alpha) \neq 0$.
- 16. If f and g are two linear functionals on a vactor space V then prove that g is a scalar multiple of f if and only if the null space of g contains the null space of f.
- 17. Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ Find the dimension of the space of characteristic

vectors associated with the characteristic value 1.

- 18. Let V be a finite dimensional vector space over the field F. Let F be a commuting family of triangulable linear operators on V. Prove that there exists an ordered basis for V such that every operator in F is represented by a diagonal matrix in that basis.
- 19. Let T be a linear operator on the finite dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Prove that there exists a diagonalizable operator D on V and a nilpotent operator N on V such that T =D+N and DN=ND.



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20. If α is a vector and β is a linear combination of an orthogonal sequence of non-zero vectors $\alpha_1, \alpha_2, \dots, \alpha_n$, then express β as a linear combination of the vectors $\alpha_1, \alpha_2, \dots, \alpha_n$.

(3)

- 21. Explain Gram Schmidt orthogonalization process with the help of a suitable example.
- 22. Prove or disprove "If V is han innerproduct space, and f a linear functional on V then there exists a unique vector β in V such that $f(\alpha) = (\alpha \mid \beta)$ for all α in ν ".
- 23. Prove that for each B in GL(n), there exists unique matrices N and U such that N is in $T^+(n)$, U is in U(n) and B=N.U.
- 24. Let V and W be finite dimensional innerproduct spaces over the same field. Then prove that V and W are isomorphic if and only if they have the same dimension.
- 25. Let V be a finite-dimensional vector space and let W_1 be any subspace of V. Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$.
- 26. Let E be a projection of V and let T be a linear operator on V. Prove that the range of E is invariant under T if and only if ETE = TE.

SECTION - D

Answer any 2 out of 4. Each question carries 6 marks. (2×6=12)

- 27. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Prove that T s diagonalizable if and only if the minimal polynomial for T has the form p = (x c₁)...(x c_k), where c₁....c_k are distinct elements of F.
- 28. Let T be a linear operator on a finite dimensional space V. If T is diagonalizable and if c_1, \ldots, c_k are the distinct characteristic values of T, Prove that there exists linear operators E_1, \ldots, E_k on V such that $I = c_1 E_1 + \ldots + c_k E_k$.

P.T.O.