29. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Prove the following.

   a) I - E is the orthogonal projection of V on $W^{\perp}$

   b) I-E is an idempotent linear transformation of V onto $W^{\perp}$ with null space W.

30. Prove that for every in vertible complex n×n matrix B there exists a unique lower triangular matrix M with positive entries on the main diagonal such that MB is unitary.

Reg. No. : ....

Name : ....

III Semester B.Sc.Hon's(Mathematics) Degree (Reg./Supple./ Improv.) Examination, November 2019

(2016 Admission onwards)

BHM 305: ADVANCED LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION - A

Answer any **4** questions out of 5. Each question carries **One** mark. **(4×1=4)**

1. Define nilpotent operator.
2. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined $by\ T(x,y)=(x+y,0)$. Find null space of T.
3. Define projection of a vector space. Give an example.
4. Is it true that "every matrix A such that A^2=A is diagonalizable."
5. Give an example of an orthogonal matrix which is not unitary.

SECTION - B

Answer any **6** out of 9.Each question carries **2** marks. **(6×2=12)**

6. Define self adjoint operator. Give an example.
7. If S is any subset of a Finite dimensional vector space V, then prove that $(S^0)^0$ is the subspace spanned by S.
8. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Prove that the null space of T^t is the annihilator of the range of T.
9. Let $A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$. Find the minimal polynomial of the operator corresponding to the matrix A.
10. Let V be a vector space and (I) an inner product on V. Show that $(0\,|\,\beta)=0$ for all β in V.

11. Find the condition that the real matrix $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ is unitary.

12. Let *V* be a finite - dimensional innerproduct space, and *T a linear operatior on V*. Suppose *W* is a subspace of *V* which is invariant under *T*. Prove that the orthogonal compliment of *W* is invariant under T^*

13. Let V be a finite dimensional inner product space. If T is a linear operator on V, show that $(T^*)^*=T$.

14. Differentiate between orthonormal set and orthogonal basis with the help of suitable examples.

SECTION - C

Answer any **8** questions out of 12. Each question carries **4** marks. **(8×4=32)**

15. If V is a finite dimensional vector space and $\alpha(\neq 0)\in V$, then prove that there exists a linear functional f such that $f(\alpha)\neq 0$.

16. If f and g are two linear functionals on a vactor space *V* then prove that g is a scalar multiple of f if and only if the null space of *g* contains the null space of *f*.

17. Let $A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ Find the dimension of the space of characteristic vectors associated with the characteristic value 1.

18. Let *V* be a finite dimensional vector space over the field F. Let $\mathscr{F}$ be a commuting family of triangulable linear operators on V. Prove that there exists an ordered basis for V such that every operator in $\mathscr{F}$ is represented by a diagonal matrix in that basis.

19. Let T be a linear operator on the finite dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Prove that there exists a diagonalizable operator D on V and a nilpotent operator N on V such that T =D+N and DN=ND.

20. If α is a vector and β is a linear combination of an orthogonal sequence of non -zero vectors $\alpha_1,\alpha_2....\alpha_n$, then express β as a linear combination of the vectors $\alpha_1,\alpha_2....\alpha_n$,.

21. Explain Gram - Schmidt orthogonalization process with the help of a suitable example.

22. Prove or disprove "If V is han innerproduct space, and f a linear functional on V then there exists a unique vector β in V such that $f(\alpha)=(\alpha\mid\beta)$ *for all* α *in* v".

23. Prove that for each B in GL(*n*), there exists unique matrices *N* and *U* such that *N* is in $T^+(n), U$ *is in* $U(n)$ and B=N.U.

24. Let V and W be finite dimensional innerproduct spaces over the same field. Then prove that V and W are isomorphic if and only if they have the same dimension.

25. Let V be a finite-dimensional vector space and let W_1 be any subspace of V. Prove that there is a subspace W_2 of *V* such that $V = W_1 \oplus W_2$.

26. Let E be a projection of V and let T be a linear operator on V. Prove that the range of E is invariant under T if and only if ETE = TE.

SECTION - D

Answer any **2** out of 4. Each question carries **6** marks. **(2×6=12)**

27. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Prove that T s diagonalizable if and only if the minimal polynomial for T has the form $p=(x-c_1)...(x-c_k)$, *where* $c_1....c_k$ are distinct elements of F.

28. Let T be a linear operator on a finite dimensional space *V*. If *T* is diagonalizable and if $c_1.....c_k$ are the distinct characteristic values of *T*, Prove that there exists linear operators $E_1,.... E_k$ on *V* such that $I = c_1E_1+.... + c_kE_k$.

P.T.O.