28. Find E (Y_1, Y_2) if Y_1 and Y_2 have joint p.d.f. given by

$$f(y_1, y_2) = \begin{cases} 2(1-y_1), 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0, \text{elsewhere} \end{cases}$$

29. Find the covariance of Y_1 and Y_2 if Y_1 and Y_2 have the joint p.d.f. given by

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0, \text{elsewhere} \end{cases}$$

30. If Y_1 and Y_2 are independent random variables and g (Y_1) and h (Y_2) are functions of only Y_1 and Y_2 respectively, show that E $[g(y_1)\ h(y_2)] = E[g(y_1)]\ [E[h(y_2)]$.

31. If Y has the probability density function $f_y(y) = \begin{cases} 2y, 0 \le y \le 1 \\ 0, \text{elsewhere} \end{cases}$, find the density function of $U = -4y + 3$.

32. If $Y_1, Y_2, \ldots, Y_n$ is a random sample of size n from a normal population with mean μ and variance σ^2, show that $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is normally distributed with mean μ and variance σ^2/n.

33. If $Z_1, Z_2, \ldots, Z_6$ denotes a random sample from the standard normal distribution, find a number b such that $P\left(\sum_{i=1}^{6} z_i^2 \le b\right) = 0.95$.

Answer **any one** essay question out of **2** : **(1×10=10)**

34. If Y_1, Y_2 and Y_3 are random variables where $E(Y_1) = 1$, $E(Y_2) = 2$, $E(Y_3) = -1$, $V(Y_1) = 1$, $V(Y_2) = 3$, $V(Y_3) = 5$, Cov $(Y_1, Y_2) = -0.4$, Cov $(Y_1, Y_3) = 0.5$ and Cov $(Y_2, Y_3) = 2$.

Find the expected value and variance of $U = Y_1 - 2Y_2 + Y_3$. If $W = 3Y_1 + Y_2$, find Cov (U, W).

35. State and prove the Central Limit Theorem.

K16U 1344

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improve.) Examination, May 2016
BHM 205 : PROBABILITY THEORY – II

Time : 3 Hours Max. Marks : 80

Answer **all 10** questions : **(10×1=10)**

1. If Y_1 and Y_2 are two random variables, define the joint distribution function $F(y_1, y_2)$.
2. If Y_1 and Y_2 are two continuous random variables, give the marginal density functions of Y_1 and Y_2.
3. Define the conditional discrete probability distribution of two discrete random variables Y_1 and Y_2.
4. Define the covariance of two random variables Y_1 and Y_2.
5. If Y_1 and Y_2 are independent random variables, show that cov $(Y_1, Y_2) = 0$.
6. If Y_1 and Y_2 are two jointly continuous random variables, define the conditional density of Y_1 given $Y_2 = Y_2$.
7. Find $P(Y_1 \ge 2 | Y_2 = 1)$ from the following table.

y_2 \ y_1	0	1	2
0	0	3/15	3/15
1	2/15	6/15	0
2	1/15	0	0

8. If Y_1 and Y_2 are jointly discrete random variables, define the conditional expectation of g (Y_1) given that $Y_2 = y_2$.
9. Define a 't' distribution with n degrees of freedom.
10. Define a χ^2 distribution with n degrees of freedom.

Answer **any 10** short answer questions out of **14** : **(10×3=30)**

11. The joint p.d.f. of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30\, y_1 y_2^2, y_1 - 1 \le y_2 \le 1 - y_1, 0 \le y_1 \le 1 \\ 0, \text{elsewhere} \end{cases}, \text{find } p(Y_1 > Y_2)$$

12. If $f(y_1, y_2) = \begin{cases} 2, 0 \le y_2 \le y_1 \le 1 \\ 0, \text{elsewhere} \end{cases}$ is the joint probability density function of the random variables Y_1 and Y_2, show that Y_1 and Y_2 are independent.

13. Find the expected value of Y_1, if the joint p.d.f. of the random variables Y_1 and Y_2 given by

$$f(y_1, y_2) = \begin{cases} 2y_1, & 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

14. If Y_1 and Y_2 are independent random variables, show that

$\text{cov}(Y_1, Y_2) = E(Y_1Y_2)E(Y_1) - E(Y_2)$.

15. If $Y_1, Y_2, \ldots, Y_n$ are independent random variables with moment generating function $m_{y_i}(t)$, i = 1, 2, ..., n, respectively, prove that $m_{y_1+y_2+\ldots+y_n}(t) = m_{y_1}(t).m_{y_2}(t) \ldots m_{y_n}(t)$.

16. If $Y_1, Y_2, \ldots, Y_n$ are independently distributed random variables with $E(Y_i) = \mu_i$, $V(y_i) = \sigma^2$, i = 1, 2, ... n and if $Z_i = \frac{Y_i - \mu_i}{\sigma i}$, i = 1, 2, ..., n, show that $\sum_{i=1}^{n} Z_i^2$ has a χ^2 distribution with n degrees of freedom.

17. Find $P(0.1 \le Y_1 \le 0.3, 0 \le Y_2 \le 0.5)$ if the joint density function of Y_1 and Y_2 is

$$f(y_1, y_2) = \begin{cases} 1, & 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

18. Contracts for two construction jobs are randomly assigned to one or more of three firms, A, B and C. If Y_1 is the number of contracts assigned to firm A and Y_2 is the number of contracts assigned to firm B, find F (1, 0).

19. Let $Y_1, Y_2, \ldots, Y_n$ be independent and identically distributed random variables such that for $0 < p < 1$, $P(Y_i = 1)$ p and $P(Y_i = 0) = q = 1 - p$. Find the moment generating function for Y_1.

20. If $Y_1, Y_2, \ldots, Y_n$ are independent, normal random variables each with mean μ and variance σ^2. Find $P\left(\left|\bar{Y} - \mu\right| \le 1\right)$ if $\sigma^2 = 16$ and n = 25.

21. If the joint p.d.f. of Y_1 and Y_2 is given by $f(y_1, y_2) = \begin{cases} 3y_1, 0 \le y_2 \le y_1 \le 1 \\ 0, \text{elsewhere} \end{cases}$, find the marginal density function for Y_2.

22. If the joint p.d.f. of Y_1 and Y_2 is given by $f(y_1, y_2) = \begin{cases} e^{-(y_1 + y_2)}, y_1 > 0, y_2 > 0 \\ 0, \text{elsewhere} \end{cases}$, examine whether Y_1 and Y_2 are dependent.

23. If $f(y_1, y_2) = \begin{cases} 6(1 - y_2), 0 \le y_1 \le y_2 \le 1 \\ 0, \text{elsewhere} \end{cases}$ in the joint p.d.f. of Y_1 and Y_2, find $V(Y_1)$ and $V(y_2)$.

24. If Y has the p.d.f. given by $f_y(y) = \begin{cases} \frac{y+1}{2}, -1 \le y \le 1 \\ 0, \text{elsewhere} \end{cases}$, find the density function of $U = Y^2$.

Answer **any 6** short essay questions out of **9** : **(6×5=30)**

25. If $f(y_1, y_2) = \begin{cases} e^{-y_1}, 0 \le y_1 \le y_2 < \infty \\ 0, \text{elsewhere} \end{cases}$ is the joint p.d.f. of the random variables Y_1 and Y_2,
    a) Find the marginal density functions of Y_1 and Y_2.
    b) Find the conditional density function of Y_1 given $Y_2 = y_2$.

26. If the joint p.d.f. of Y_1 and Y_2 is given by $f(y_1, y_2) = \begin{cases} y_2, 0 \le y_1 \le y_2 \le 2 \\ 0, \text{elsewhere} \end{cases}$, find $P(Y_1 \le 1 | Y_2 = 1.5)$.

27. If $f(y_1, y_2) = \begin{cases} 6y_1y_2^2, 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0, \text{elsewhere} \end{cases}$, show that Y_1 and Y_2 are independent.