Answer **any one** essay question out of **2** : **(1×10=10)**

34. a) Show that the center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

    b) The 10 m long rod thickness from left to right so that its density, instead of being constant, is $\delta(x) = 1 + \left(\frac{x}{10}\right)$ kg/m. Find the rod's center of mass.

35. a) Find the Taylor series and Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

    b) Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} n!x^n$. Also find for what value of x does the series converge, absolutely converge and conditionally converge ?

K18U 0302

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Supple./Improv.) Examination, May 2018

BHM 203 : INTEGRAL CALCULUS

(2013-15 Admns.)

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Find the derivative of y w.r.t. x where $y = \ln (x^2 + 3)$.
2. Show that for every real number x, $e^x = \ln^{-1} x$.
3. Give an example of a sequence having no upper bound.
4. By defining the recursion formula, find the Fibonacci numbers.
5. Establish the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{4^n}$.
6. Evaluate $\sum_{k=1}^{4} (k^2 - 3k)$.
7. Give an example of a function with no Riemann integral.
8. Find the norm of the partition $p = \{0, 0.2, 0.6, 1, 1.5, 2\}$ of $[0, 2]$.
9. Applying the fundamental theorem of calculus, find $\frac{d}{dx}\int_{-\pi}^{x} \cos t \, dt$.
10. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta \, d\theta}{3 + 2\sin\theta}$.

Answer **any 10** short answer questions out of 14 . **(10×3=30)**

11. Find $\frac{dy}{dx}$ if $y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}$, $x > 1$.

12. Find K if $e^{2K} = 10$.

13. Solve the initial value problem, $e^y \frac{dy}{dx} = 2x,\ x > \sqrt{3},\ y(2) = 0$.

14. Evaluate $\int \frac{\log_2 x\, dx}{x}$.

15. Find $\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - \left(x/2\right)}{x^2}$.

16. Show that $\sqrt{x^2+5}$ and $\left(2\sqrt{x}-1\right)^2$ grow at the same rate at $x \to \infty$.

17. Find the sum $\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}}$.

18. Find the length of the curve $y = \left(x/2\right)^{2/3}$ from $x = 0$ to $x = 2$.

19. Find the Maclaurin series for the function $f(x) = \sin x$.

20. Does the sequence whose n^{th} term given by $a_n = \left(\frac{n+1}{n-1}\right)^n$ converge ? If so, find $\lim_{n \to \infty} a_n$.

21. Evaluate $\int_0^1 \frac{2dx}{\sqrt{3+4x^2}}$.

22. For what values of x the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$ converges.

23. Find the workdone by a force of $f(x) = \frac{1}{x^2}$ N along the x-axis from x = 1 m to x = 10 m.

24. Find the area of the region in the plane enclosed by the cardioid $r = 2(1+\cos\theta)$.

Answer **any 6** short essay questions out of **9**. **(6×5=30)**

25. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a = 2$. Where does the series converges to $\frac{1}{x}$?

26. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t\, dt$, using Fundamental Theorem.

27. Find the length of the cardioid $r = 1 - \cos\theta$.

28. Find the area of the region in the first quadrant that is bounded by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

29. Find the area of the region between the curve $y = 4 - x^2$, $0 \le x \le 3$ and the x-axis.

30. Show that if f is continuous on [a, b], $a \neq b$ and if $\int_a^b f(x) = 0$, then $f(x) = 0$ atleast once in [a, b].

31. Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and the lines $y = 0$, $x = 2$ about the x-axis.

32. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis.

33. Evaluate : $\int_0^{\ln 2} 4e^x \sinh x\, dx$.