Answer **any one** essay question out of **2**. **(1×10=10)**

34. If f is continuous on [a, b], prove that $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point of [a, b] and $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x), a \le x \le b$.

35. Find the centre of mass of a thin plate of constant density ρ covering the region bounded above by the parabola $y = 4 - x^2$ and below the x axis.

K16U 1342

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improve.) Examination, May 2016
BHM 203 : INTEGRAL CALCULUS

Time : 3 Hours Max. Marks : 80

Answer **all** the **10** questions. **(10×1=10)**

1. Find K if $e^{2K} = 10$.
2. Find $\int_2^4 \frac{dx}{x\,(ln x)^2}$.
3. Evaluate $\int \frac{\log_2^x}{x}\,dx$.
4. Find the limit of the sequence whose n^{th} term is $\frac{\cos^n}{n}$.
5. For what values of x, the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges.
6. Obtain the Maclaurin's series for sin x.
7. Evaluate $\sum_{k=1}^{3} (-1)^{K+1} \sin(\pi/k)$.
8. Find the average value of $f(x) = 4 - x^2$ on [0, 3].
9. Find the work done by a force of $F(x) = \frac{1}{x^2}N$ along the x axis from x = 1 m to x = 10 m.
10. The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$ and the x axis is revolved about the x axis to generate a solid. Find the volume of the solid generated.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. If an amount A_0 is deposited at a fixed annual interest rate r% and if the interest is added to the account K times a year, prove that the amount at the end of t years is $A_0 e^{rt}$, where r is expressed in decimal.

12. Find $\lim_{x \to 0} \left(\frac{1 - \cos x}{x + x^2} \right)$.

13. Show that $x + \sin x = O(x)$.

14. Find $\int_0^{ln 2} 4e^x \sin hx \, dx$.

15. Show that the sequence $\left\{ \frac{1}{n} \right\}$ converges to zero.

16. If f(x) is a function defined for all $x > n_0$ and that $\{a_n\}$ is a sequence of real numbers such that $a_n = f(n)$ for $n \geq n_0$, show that $\lim_{x \to \infty} f(x) = L$ implies $\lim_{n \to \infty} a_n = L$.

17. Show that the geometric series $a + ar + \ldots\ldots + ar^{n-1} + \ldots\ldots$ converges if $|r| < 1$ and diverges if $|r| > 1$.

18. Find the Taylor series and Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$.

19. Show that the constant function is Riemann integrable over [a, b].

20. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

21. Evaluate $\int_0^{\pi} 5 (5 - 4 \cos t)^{1/4} \sin t \, dt$.

22. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$; the lines $y = 1$, $x = 4$ about the line $y = 1$.

23. Find the centre of mass of a wire of constant density ρ shaped like a semi circle of radius 'a'.

24. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

Answer **any 6** short essay questions out of **9**. **(6×5=30)**

25. Solve the initial value problem $\frac{d^2y}{dx^2} = 2e^{-x}$, $y(0) = 1$, $y'(0) = 0$.

26. Find $\lim_{x \to \infty} x^{1/x}$.

27. Show that $\sin h^{-1} x = ln \left[x + \sqrt{x^2 - 1} \right], -\infty < x < \infty$.

28. Show that $\lim_{n \to \infty} \left(\frac{ln}{n} \right) = 0$.

29. Find the Taylor series generated by $f(x) = \frac{1}{x}$ at the point $a = 2$.

30. Using the definition of definite integral, prove that $\int_a^b k f(x) \, dx = k \int_a^b f(x) dx$.

31. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 1$.

32. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

33. Find the length of the cardioid $r = 1 + \cos\theta$.