30. If min f and max f are the minimum and maximum values of f on [a, b], using Riemann's sums prove that $\min f \cdot (b-a) \leq \int_a^b f(x)\,dx \leq \max f \cdot (b-a)$.

31. Find the area of the region between the x axis and the graph of $f(x) = x^3 - x^2 - 2x,\ -1 \leq x \leq 2$.

32. Find the length of the plane curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1,\ 0 \leq x \leq 1$.

33. Find the length of the cardioid $r = 2\,(1 - \cos\theta)$.

Answer **any one** essay question out of **2** : **(1×10=10)**

34. Find the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq \frac{1}{2}$ about the x axis.

35. If f is continuous on [a, b], then prove that $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point of [a, b] and $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x), a \leq x \leq b$.

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Reg. No. : ....

Name : ....

II Semester B.Sc. (Hon's) Degree (Mathematics – Reg./Supple./Improv.) Examination, May 2015
BHM 203 : INTEGRAL CALCULUS

Time: 3 Hours Max. Marks : 80

Answer **all** the **10** questions : **(10×1=10)**

1. Define the natural logarithm function and interpret it geometrically.

2. Find $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3 + 2\sin\theta}\,d\theta$.

3. Find $\frac{dy}{dx}$ if $y = x^x$, $x > 0$.

4. Find the limit of the sequence $\left\{\frac{\cos n}{n}\right\}$

5. For what values of x the series $\sum_{n=1}^{\infty} (-1)^n . \frac{x^n}{n}$ converges.

6. Obtain the Maclaurins's series for cos x.

7. Evaluate $\sum_{K=1}^{4} \cos K\pi$.

8. Find the average value of $f(x) = 4 - x^2$ on [0, 3].

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9. Find the work done by a force of $F(x) = \frac{1}{x^2}$ N along the x axis from $x = 1$ m to $x = 10$ m.

10. Find the volume of the solid generated by revolving the region between the curve $y = \sqrt{x}$, $0 \le x \le 4$ and the x axis about the x axis.

Answer **any 10** short answer questions out of **14** : **(10×3=30)**

11. Prove that the solution of the initial value problem $\frac{dy}{dt} = ky$, with $y = y_0$ when $t = t_0$ is the law of exponential change.

12. Find $\lim_{x \to 0} \frac{x - \text{Sin}\,x}{x^3}$.

13. Show that $e^x + x^2 = 0(e^x)$.

14. Evaluate $\int_0^1 \text{Sinh}^2 x\,dx$.

15. Show that the constant sequence {K} converges to K.

16. If f(x) is a function defined for all $x \ge n_0$ and $\{a_n\}$ is a sequence of real numbers such that $a_n = f(n)$ for $n \ge n_0$, then show that $\lim_{x \to a} f(x) = L$ implies $\lim_{x \to \alpha} a_n = L$.

17. If a ball is dropped from 'a' metres above a flat surface and each time the ball hits the surface after falling a distance h, it rebounds a distance rh, where $0 < r < 1$, find the total distance the ball travels up and down.

18. Find the Taylor series and Taylor polynomial generated by f(x) = Cosx at x = 0.

19. If $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$, show that f is not Riemann integrable.

20. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \text{Cos}\,t\,dt$.

21. Evaluate $\int_{\pi/4}^{\pi/2} \text{Cot}\,\theta\,\text{csc}^2\,\theta\,d\theta$.

22. A pyramid 3 m high has a squarebase that is 3 m on a side. The cross section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

23. Show that the centre of mass of a straight, thin strip or rod of constant density lies half way between the two ends.

24. Find the area of the region that lies inside the circle r = 1 and outside the cardioid $r = 1 - \text{Cos}\,\theta$.

Answer **any 6** short essay questions out of **9** : **(6×5=30)**

25. Solve the initial value problem $e^y \frac{dy}{dx} = 2x, x > \sqrt{3}, y(2) = 0$.

26. Find $\lim_{x \to \theta} \left(\frac{1}{\text{Sin}x} - \frac{1}{x} \right)$.

27. Show that $\text{Cosh}^{-1} x = l_n \left(x + \sqrt{x^2 - 1} \right), x \ge 1$.

28. Does the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1} \right)^n$ converge ? If so find $\lim_{x \to \infty} a_n$.

29. If $\sum a_n = A$ and $\sum b_n = B$ are convergent series, show that $\sum (a_n + b_n)$ converges and converges to A + B.