32. Find the area of the surface generated by revolving about the x-axis, the arc of the parabola $y^2 = 4ax$ from the origin to the point where $x = a$, $a > 0$.

33. The ends of two thin steel rods of equal length are welded together to make a right angled frame. Locate the centre of mass of the frame.

Answer **any one** essay question of 2 :   **(1×10=10)**

34. State and prove the convergence theorem for power series.

35. a) How much work does it take to pump the water from a full upright circular cylindrical tank of radius 5 m and height 10 m to level of 4 m above the top of the tank ?

   b) Show that the center of mass of a straight thin strip or rod of constant density lies half-way between the two ends.

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Time : 3 Hours   Max. Marks : 80

Answer **all** the **ten** questions :   **(10×1=10)**

1. Find the derivative of y with respect to x, where $y = e^{(2x + 5)}$.

2. Evaluate $\int_0^2 \frac{2x}{x^2 - 1}\, dx$.

3. Find the derivative of $y = 3^{\sin x}$ with respect to x.

4. Define the recursion formula for Fibonacci numbers.

5. Establish the convergence of the series $\sum_{n=1}^{\infty} 5\left(\frac{-1}{4}\right)^n$.

6. Evaluate $\lim_{x \to \infty} \frac{x - 2}{x^2 - 4}$.

7. Describe the monotonicity of Riemann integrals.

8. Applying the fundamental theorem of calculus. Find $\frac{d}{dx} \int_{-\pi}^{\pi} \cos t\, dt$.

9. Write the expression for the work done W by a force F.

10. What is the formula for the area of the surface generated by revolving a curve $y = f(x)$, which is smooth on [a, b], around x-axis ?

Answer **any 10** short answer questions out of 14 :   (10×3=30)

11. Find $\frac{dy}{dx}$ if $y = \frac{(x^2 + 1)(x + 1)}{x - 1}$.

12. Find the value of k if $e^{2k} = 10$.

13. Evaluate $\int \frac{\log_2 x}{x}\, dx$.

14. Investigate the convergence of the series $\sum\limits_{n=1}^{\infty} \frac{n^n}{n!}$.

15. Show that the $\left\{ (-1)^{n+1} \frac{(n-1)}{n} \right\}$ diverges.

16. For what values of x, does the power series $\sum\limits_{n=1}^{\infty} \frac{x^n}{n}$ converge ?

17. Find the Taylor series generated by the function $f(x) = \frac{1}{x}$ at x = 2.

18. Does the sequence whose n^{th} term given by $a_n = \left(\frac{n+1}{n-1} \right)^n$ converge ? If

    so find $\lim\limits_{n \to \infty} a_n$.

19. Find the average value of $f(x) = 4 - x^2$ on [0, 3]. Explain whether f actually takes on this value at some point in the given domain.

20. Evaluate $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot\theta \cos^2\theta\, d\theta$.

21. Describe the fundamental theorem on calculus and the mean value theorem for definite integrals.

22. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

23. Evaluate $\int\limits_{-\infty}^{\infty} \frac{dx}{1 + x^2}$.

24. Find the length of the curve y = log sec x between the points given by x = 0 and $x = \frac{\pi}{3}$.

Answer **any 6** short answer questions out of 9 :   (6×5=30)

25. People who do carbon-14 dating use a figure of 5700 years for its half life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

26. Determine $\lim\limits_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

27. Find the Taylor series and the Taylor polynomial generated by the function $f(x) = e^x$ at x = 0.

28. Obtain the power series for $\frac{1}{(1-x)^2}$ by multiplying the geometric series

    $\sum\limits_{n=0}^{\infty} x^n, |x| < 1$.

29. If f is continuous on [a, b], a ≠ b and if $\int\limits_{a}^{b} f(x) = 0$, then show that f(x) = 0 at least once in [a, b].

30. Find the area enclosed by the curves $x = y^2$ and x = y + 2.

31. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at an angle of 45° angle at the center of the cylinder. Find the volume of the wedge.