K20U 0233

Reg. No. : ...............................

Name : ...................................

**Second Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, April 2020**

**(2016 Admission Onwards)**

**BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA**

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Give an example of an infinite non abelian group.

2. Find the number of elements in the cyclic subgroup generated by 25 of $\mathbb{Z}_{30}$.

3. Verify whether {(1, 2), (2, 1)} is a basis for $\mathbb{R}^2$.

4. Let V be a vector space of all functions f from $\mathbb{R}$ into $\mathbb{R}$. Verify whether $\{f \in V : f(-1) = 0\}$ is a subspace.

5. Find two linear operators U and T such that UT = 0 but TU $\neq$ 0. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Show that a group G is abelian if and only if $(a * b)^2 = a^2 * b^2$, $\forall$ a, b $\in$ G.

7. Find the product $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 5 & 2 & 1 \end{pmatrix}$.

8. Prove that every permutation in $S_n$, $n \geq 2$ can be expressed as a product of transpositions.

9. What is the maximum possible order of an element in $A_{10}$.

10. Write all the generators of $\mathbb{Z}_2 \times \mathbb{Z}_3$.

P.T.O.

11. Prove that the sum of two subspaces is a subspaces.

12. Find the dimension of the vector space of all $2 \times 2$ real symmetric matrices.

13. True or false : there is a linear operator T and $\mathbb{R}^2$ such that $T(1, 1) = (2, 2)$ and $T(2, 2) = (1, 1)$. Justify.

14. Prove that the composition of two linear operators is linear. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Prove or disprove : $\left\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R}\right\}$ is an abelian group under matrix multiplication.

16. Write all the elements of the cyclic subgroup generated by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ in $GL(2, \mathbb{R})$.

17. Prove that every even permutation of $S_n$, $n > 2$ can be expressed as a product of 3-cycles.

18. Find $\sigma^{100}$ where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$.

19. Prove that every subgroup of a cyclic group is cyclic.

20. Let p and q be two distinct primes. Find the number of generators of the cyclic group $\mathbb{Z}_{pq}$.

21. Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$; let $V_e$ be the set of even functions, let $V_o$ be the subset of odd functions. Prove that $V_e$ and $V_o$ are subspaces of V.

22. Find the number of one dimensional subspaces of a 3-dimensional vector space over the field of 5 elements.

23. Prove that the dimension of the vector space of all real polynomials is not finite.

24. If $W_1$ and $W_2$ are two subspaces of a vector space V, prove that $W_1 \cup W_2$ is a subspace if and only if one is contained in the other.



25. Let T be an invertible linear transformation from V on to W. Prove that $T^{-1}$ is linear.

26. Prove that every n-dimensional vector space over the field $\mathbb{F}$ is isomorphic to $\mathbb{F}^n$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Let G be a group and 'a' is one fixed element of G. Prove that $H_a = \{x \in G : ax = xa\}$ is a subgroup of G.

28. State and prove Cayley's theorem.

29. Let V be an n-dimensional vector space over the field $\mathbb{F}$ and let $\mathcal{B}$ and $\mathcal{B}'$ be two ordered bases of V. Prove that there is a unique, necessarily invertible, $n \times n$ matrix P with entries in $\mathbb{F}$ such that $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$.

30. Let A and B be the matrix representations of the linear operators T and U respectively with respect to the basis $\mathcal{B}$. Prove that the matrix of TU with respect to the basis $\mathcal{B}$ is AB. **(2×6=12)**