K18U 0305

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.) Examination, May 2018
(2016 Admn. Onwards)
BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of **5** questions. **Each** question carries **1** mark.

1. State True/False : The set of all non-zero rational numbers is closed under the operation division.
2. What is the cyclic subgroup of $\langle \mathbb{Z}, + \rangle$ generated by 2.
3. What is the dimension of $\mathbb{C}$ over the field $\mathbb{R}$?
4. What do you mean by an ordered basis for a finite dimensional vector space ?
5. Is the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (1 + x_1, x_2)$ linear ? **(4×1=4)**

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks.

6. Define an abelian group. Give an example for a group which is not abelian.
7. Prove that in a group, $(a * b)^{-1} = b^{-1} * a^{-1}$.
8. Give examples of two groups of order 4, one of which is cyclic and the other is non-cyclic.

P.T.O.

9. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ are two permutations on S_6, find $\tau\sigma$.

10. Define the terms orbits, cycles and transpositions in a permutation.

11. Define subspace of a vector space. Give an example for the subspace of $\mathbb{R}^2$.

12. Define basis and dimension of a vector space.

13. What do you mean by rank and nullity of a linear transformation ?

14. Define an invertible function between two vector spaces. Is the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$ invertible ? **(6×2=12)**

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks :

15. Let S be the set of all real numbers except – 1. Define $*$ on S by $a * b = a + b + ab$. Prove that S is an abelian group with respect to $*$.

16. Prove that intersection of two subgroups is again a subgroup. By an example, show that union of two subgroups need not be a subgroup.

17. Find all subgroups of Z_{18} and draw their subgroup diagram.

18. Describe the elements of S_3, the group of symmetries of an equilateral triangle.

19. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ in S_8 as a product of disjoint cycles and then as a product of transpositions.

20. Prove that every permutation of a finite set can be expressed as a product of disjoint cycles.

21. Prove that a non-empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar c in F, the vector $c\alpha + \beta$ is again in W.

22. Let W be a subspace of a finite dimensional vector space V. Prove that every linearly independent subset of W is finite and is part of a basis of W.

23. Are the vectors {(1, 1, 2, 4), (2, –1, –5, 2), (1, –1, – 4, 0), (2. 1, 1, 6)} linearly independent in $\mathbb{R}^4$?

24. Find the rank and nullity of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (2x_1 + x_2, x_2 - x_1, 3x_1 + 4x_2)$.

25. Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

26. Let T be a linear operator on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (-x_2, x_1)$. What is the matrix of T in the ordered basis {(1, 2), (1, –1)} ? **(8×4=32)**

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. a) Prove that identity element in a group is unique.

b) Show that a nonempty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$ for all a, b, $\in$ H.

28. State and prove Cayley's theorem.

29. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.

30. Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space over F. Prove that the space of all linear transformations from V to W, L(V, W) is finite dimensional and has dimension mn. **(2×6=12)**
