Reg. No.: ....

Name: ....

K19U 0771

II Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supp./Imp.) Examination, April 2019
(2016 Admission Onwards)
BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

(Answer **any 4** questions out of **5** questions. **Each** question carries **1** mark).

1. What is the number of commutative binary operation on a set with n elements ?
2. Write all the subgroups of S_3.
3. Give an example of finite vector space.
4. Write a basis for the vector space of all real 2×2 matrices.
5. Define rank of a linear transformation. **(4×1=4)**

SECTION – B

(Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks).

6. Prove that the identity of a group and the inverse of an element are unique.
7. Write $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 5 & 2 & 1 \end{pmatrix}$ as a product of disjoint cycles.
8. Prove that every cycle of length r can be expressed as product of r – 1 transpositions.
9. What is the maximum possible order of an element in S_{10} ?
10. Prove that every cyclic group is abelian.

P.T.O.

11. Prove that the intersection of two subspaces is a subspace.

12. Show that the set of vectors {(1, 1, 0, 0), (0, 0, 1, 1), (1, 0, 0, 4), (0, 0, 0, 2)} is linearly independent subset of $\mathbb{R}^4$.

13. Show that T(x, y, z) = (x + 1, 2y, 3z) is not a linear operator on $\mathbb{R}^3$.

14. Prove or disprove : there is no linear transformation from $\mathbb{R}^2$ on to $\mathbb{R}^3$.

(6×2=12)

SECTION – C

(Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks).

15. True or false : If all proper subgroups of a group are cyclic, then the group is cyclic. Justify.

16. Show that GL(2, $\mathbb{R}$) is a group under matrix multiplication.

17. Find the number of distinct r-cycles in S_n.

18. Compute the cyclic subgroup generated by $\sigma = (1, 2, 3)(1, 2, 4)$ in S_4.

19. Prove that A_n contains exactly n!/2 elements for $n \geq 2$.

20. Prove that S_n is non abelian for $n \geq 3$.

21. Let V be a finite dimensional vector space. Prove that every linearly independent subset of V is a part of a basis.

22. Find the number of one dimensional subspaces of a 2-dimensional vector space over the field of 3 elements.

23. Let V be a vector space over the field $\mathbb{F}$. Suppose there are a finite number of vectors $\alpha_1, ..., \alpha_r$ in V which span V. Prove that V is finite-dimensional.

24. Prove that $\mathbb{R}$ is not a finite dimensional vector space over $\mathbb{Q}$.

25. Prove that for an m × n matrix A with entries in the field F, column rank of A and row rank of A are same.

26. Prove that the nullspace of a linear transformation is a subspace.

(8×4=32)

SECTION – D

(Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks).

27. Prove or disprove : $\left\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R} \setminus \{0\}\right\}$ is an abelian group under matrix multiplication.

28. Show that for every subgroup H of S_n for $n \geq 2$, either all permutations in H are even or exactly half of them are even.

29. Let W_1 and W_2 are two subspaces of a finite dimensional vector space V. Prove that $\dim(W_1 + W_2) + \dim(W_1 \cap W_2) = \dim W_1 + \dim W_2$.

30. State and prove rank nullity theorem. **(2×6=12)**
