K18U 0301

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Supple./Improv.) Examination, May 2018
(2013-15 Admns.)
BHM 202 : ABSTRACT ALGEBRA – I

Time : 3 Hours

Max. Marks : 80

Answer **all** the questions : **(10×1=10)**

1. Prove that $\langle \mathbb{Z}, + \rangle$ is isomorphic to $\langle 2\mathbb{Z}, + \rangle$.

2. On $\mathbb{Z}$, let $H = \{n^2, n \in \mathbb{Z}^+\}$. Determine whether H is closed under
   a) Usual addition and
   b) Multiplication.

3. In $\mathbb{Z}_4$, {0, 3} is not a subgroup. Justify.

4. What are the generators of $\mathbb{Z}$ under addition ?

5. Define a transposition.

6. State true/false :
   Every permutation can be expressed both as a product of an even number of transpositions or an odd number of transpositions.

7. Define group homomorphism.

8. Define Kernal of a homomorphism.

9. Define an integral domain.

10. Find the order of the cyclic subgroup of $\mathbb{Z}_4$ generated by 3.

P.T.O.

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. What are the structural properties of a binary structure ? State a few structural and non-structural properties.

12. Prove that every permutation σ of a finite set is a product of disjoint cycles.

13. Write all left and right cosets of the subgroup 3Z of Z.

14. Prove that the order of every subgroup of a group divides order of the group.

15. Prove that every group of prime order is cyclic.

16. Let S_n be a symmetric group of n elements and $\phi : S_n \to Z_2$.

    Let $\phi : S_n \to Z_2$ defined by $\phi(\sigma) = \begin{cases} 0 \text{ if } \sigma \text{ is an even permutation} \\ 1 \text{ if } \sigma \text{ is an odd permutation} \end{cases}$, show that ϕ is a homomorphism.

17. Prove that a group homomorphism $\phi : G \to G'$ is a one-to-one map if and only if $\text{Ker}\phi = \{e\}$.

18. If R is a ring with additive identity 0. Prove that (–a) (–b) = ab.

19. Prove that every field is an integral domain.

20. Define characteristic of a ring R. Find the characteristic of $\mathbb{Z}$.

21. Find the orbits of the permutation :

    $\sigma \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$.

22. Find the partition of $\mathbb{Z}_6$ into cosets of the subgroup H = {0, 3}.

23. Let σ = (1, 2, 5, 4) (2, 3) in S_5. Find the index of $\langle\sigma\rangle$ in S_5.

24. Prove that every Boolean ring is commutative.

Answer **any 6** questions out of 9 : **(6×5=30)**

25. Let $*$ be defined by $a * b = \frac{ab}{2}$ on Q^+. Prove that $(Q^+, *)$ is a group.

26. Prove that a subgroup of a cyclic group is cyclic.

27. Find all subgroups of $\mathbb{Z}_{18}$.

28. If $n \geq 2$, prove that the collection of all even permutations of {1, 2, 3, ..., n} form a subgroup of order $\frac{n!}{2}$.

29. Find all units of Z_{14}.

30. Define an idempotent element. Prove that the set of all idempotent elements of a commutative ring is closed under multiplication.

31. Prove that every finite integral domain is a field.

32. Prove that Z_2 is an integral domain. But the matrix ring $M_2(Z_2)$ has divisors of zero.

33. Prove that the map $\phi : Z \to Z_n$ where $\phi(a)$ is the remainder of a modulo n is a ring homomorphism for each positive integer n.

Answer **any one** essay question out of 2. **(1×10=10)**

34. Let G be a cyclic group with generator a. If the order of a is finite, then G is isomorphic to $\langle Z, + \rangle$. If G has finite order n, then prove that G is isomorphic to $\langle Z_n, +_n \rangle$.

35. State and prove division algorithm for $\mathbb{Z}$. Find the quotient q and remainder r when –38 is divided by 7 according to division algorithm.
