Reg. No. : ....

Name : ....

K19U 0786

II Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2019
(2013-2015 Admissions)
BHM 202 : ABSTRACT ALGEBRA – I

Time : 3 Hours   Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Define binary operation on a set.
2. Is usual substraction associative on the set of integers ? Justify.
3. Is empty set a group ? Justify.
4. Find the number of generators of Z_7.
5. Correct the statement and justify : Klein-4 group is cyclic.
6. Find the number of subgroups of Z_8.
7. Find the order of the symmetric group S_{10}.
8. State Lagrange theorem.
9. Define Kernal of a group homomorphism.
10. Give two examples of Integral Domains.

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Show that every group G with identity e and such that $x * x = e \; \forall \, x \in G$ is abelian.
12. Prove that for a group G. $(a * b)' = b' * a', \forall a, b \in G$.

P.T.O.

13. State true of false with justification : Every finite group of atmost three elements is abelian.

14. Prove that every cyclic group is abelian.

15. Show that if $H \leq G$ and $K \leq G$ then $H \cap K \leq G$.

16. Is $\mathbb{R}^+$ under operation $a * b = a - b$ a group ?

17. Define cyclic groups and give two examples.

18. Let $\phi : G \to G'$ be a group homomorphism. Prove that if H is a subgroup G. $\phi[H]$ is a subgroup of G'.

19. Determine whether the given map is a homomorphism $\phi : Z_9 \to Z_2$ given by $\phi(x)$ = remainder of x when divided by 2.

20. Prove that every group of prime order is cyclic.

21. Find the number of cosets of {0, 5} in Z_{25}.

22. Find all solution of the equation $x^2 + 2x + 2 = 0$ in Z_6.

23. Find all zero divisors of Z_5. Explain the answer.

24. Show that the intersection of subrings of a ring R is again a subring of R.

Answer **any 6** short answer questions out of **9**. **(6×5=30)**

25. Prove that left and right cancellation law hold in a group.

26. Let n be a positive integer. Let $nZ = \{nm : m \in Z\}$.

    a) Show that $\langle nZ, + \rangle$ is a group.

    b) Show that $\langle nZ, + \rangle \cong \langle Z, + \rangle$.

27. Compute the product involving the following permutations in S_6.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}, \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$$

    i) $\sigma^{-1}\tau\sigma$ ii) $\tau^2\sigma$.

28. Let G be a group and let $a \in G$. Show that $Ha = \{x \in G : xa = ax\}$ is a subgroup of G.

29. Find all subgroups of S_4 and draw its subgraph diagram.

30. Show that if a group G with identity e has finite order n. then $a^n = e$, $\forall a \in G$.

31. Compute Ker ϕ and $\phi(25)$ for $\phi : Z \to Z$ such that $\phi(1) = 4$.

32. Find the partition of Z_6 into cosets of the subgroup H = {0.3}.

33. Prove that Z_2 is an Integral domain. But the matrix ring $M_2(Z_2)$ has divisors of zero.

Answer **any one** essay question out of **2**. **(1×10=10)**

34. a) State and prove Cayley's theorem.

    b) Find the number of elements in the set $\{\sigma \in S_5 : \sigma(2) = 5\}$.

35. State and prove Lagrange's theorem.