K19U 3009

I Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.)
Examination, November - 2019
(2016 Admission Onwards)
BHM 105: TWO DIMENSIONAL GEOMETRY

Time : 3 Hours   Max. Marks : 60

SECTION-A

Any **4** out of **5** questions. Each question carries **1** mark.

1. Define translation of axes.
2. Write the general form of a homogeneous second degree equation.
3. Write the parametric equation of the parabola.
4. Define conjugate hyperbola.
5. Define centre of a conic section.

SECTION - B

Answer any **6** questions out of **9** questions. Each question carries **2** marks.

6. Find the equation of the curve $2x^2 - 3y^2 + 4x - 2y + 8 = 0$ referred to a new origin at the point (1,2) the old and new axes being parallel.

7. Let the pair of the lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair. Prove that $pq = -1$.

8. P (x, y) is equidistant from the point A (2, 0) and y– axis. Find the equation of the locus of P.

9. The normal at a point t_1 on the parabola $y^2 = 4ax$ meets it again at the point t_2. Prove that $t_2 = -t_1 - \dfrac{2}{t_1}$

10. Find the locus of the middle points of chords of the parabola $y^2 = 4ax$ which passes through the fixed point (h,k).

11. Find the condition for $lx + my + n = 0$ to be a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

12. Prove that the locus of the poles of normal chords of the rectangular hyperbola $xy = c^2$ is the curve $(x^2 - y^2)^2 + 4c^2xy = 0$.

13. Find the equation of the chord joining two points on a conic.

14. Find the asymptotes of the equation $8x^2 + 10xy - 3y^2 - 2x + 4y = 2$P.T.O.

SECTION-C

Answer any **8** questions out of 12 questions. Each questions carries 4 marks.

15. Plot the locus of the equation $x^2 + y^2 - 4x - 5 = 0$.

16. Find the coordinates of the points which divide the line joining the points $(-1, -5)$ and $(1, -2)$ internally and externally in the ratio 4 : 3.

17. What does the equation $x^2 - xy + y^2 - 6 = 0$ become when the axes are turned through an angle of $45°$

18. Find the pole of the line $lx + my + n = 0$ with respect to the parabola $y^2 = 4ax$.

19. Prove that the semi latus rectum is a harmonic mean between the segments of any focal chord of a parabola.

20. Prove that the tangents at the extremities of a focal chord of a parabola intersect at right angles on the directrix.

21. Prove that four normals can be drawn to the hyperbola $xy = c^2$ from any given point.

22. A line through the origin meets the circle $x^2 + y^2 = a^2$ at P and the hyperbola $x^2 - y^2 = a^2$ at Q. Prove that the locus of the point of intersection of the tangent at P to the circle with the tangent at Q to the hyperbola is the curve $(a^4 + 4y^4)^2 = a^6$.

23. Find the equation of the chord joining two points whose eccentric angles are given.

24. What conics do the following equation represent and find its centre $12x^2 - 23xy + 10y^2 - 25x + 26y = 14$.

25. Find the polar equation of a conic, the focus being at the pole.

26. Prove that the semi latus rectum of any conic is a harmonic mean between the segments of any focal chord.

SECTION D

Answer any **2** questions out of **4** questions. Each question carries **6** marks.

27. Prove that the general homogeneous equation of second degree represents a pair of lines passing through the origin.

28. Prove that the locus of the points such that two of the three normals to the parabola $y^2 = 4ax$ from them coincide is $27ay^2 = 4(x - 2a)^2$.

29. Find the locus of the point of intersection of tangents to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which meet at right angles.

30. Trace the conic $x^2 - 4xy - 2y^2 + 10x + 4y = 0$.

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