32. Define probability generating function. Find the probability generating function of a binomial random variable and use it to find the mean.

33. Let X and Y be two independent random variables then prove that $E(XY) = E(X)E(Y)$.

Answer **any one** essay question out of 2 : **(1×10=10)**

34. State and prove Tchebysheff's theorem and give the applications of it.

35. The length of time to failure (in hundreds of hours) for a transistor is a random variable X with distribution function given by

$F(y) = 0, \quad y < 0$

$= 1 - \exp(-y^2), \; y \geq 0.$

i) Show that F(x) has the properties of a distribution function.

ii) Find f(x).

iii) Find the probability that the transistor operates for at least 200 hours.

iv) Find $P(X > 100/X \leq 200)$.

K16U 2599

Reg. No. : ....

Name : ....

I Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2016

BHM 105 : PROBABILITY THEORY – I

Time : 3 Hours Max. Marks : 80

Answer **all** the **10** questions : **(10×1=10)**

1. Define a random variable.
2. Define expected value of a continuous random variable.
3. Define moment generating function of a random variable.
4. Define Poisson distribution.
5. Define variance of a random variable.
6. If the mean and variance of Binomial distribution are 6 and 2 respectively, find the probability density function.
7. Define Gamma distribution.
8. Define the r^{th} moment of a random variable about it's mean.
9. If $M_x(t)$ be the moment generating function of random variable X, find the moment generating function of $Y = aX + b$.
10. If X and Y are two independent random variables, then what is the co-variance between X and Y ?

Answer **any 10** short answer questions : **(10×3=30)**

11. Find the moment generating function of a Binomial distribution.
12. If X has a geometric distribution with probability of success p, show that $P(X \text{ is an odd number}) = 1/(1 + q)$.

13. Consider a line of length 5 units. Select a point randomly on the line which divides the line into two parts. Let the random variable X denote the length of first part, write the probability density function of X and find expected length of first part.

14. The random variable X has a Poisson distribution and is such that P(0) = P(1). Find P(2).

15. If the distribution function of a random variable Y is given by

$F(y) = 0$, if $y \leq 0$

$= y/8$ if $0 < y < 2$

$= y^2/16$ if $2 \leq y < 4$

$= 1$ if $y \geq 4$

Find :

i) The density function of Y

ii) $P(1 \leq Y \leq 3)$

iii) $P(Y \geq 1/Y \leq 3)$

16. Let X be a continuous random variable with density function f(x), where $x \geq 0$. If F(x) is the distribution function of X show that

$$E(X) = \int_0^\infty [1 - F(X)]dx$$

17. If X is a normal variate with mean 30 and variance 25, find

i) $P(X \geq 40)$

ii) $P(X < 25)$

iii) $P(|X| < 35)$

18. Find the moment generating function of a Gamma distribution and hence find the mean.

19. A random variable X has a uniform distribution over the interval (a, b). Derive the variance of Y.

20. Explain how moments can be derived from moment generating function.

21. Let X be a random variable with mean 10 and variance 3 find a lower bound to $P(5 < X < 15)$.

22. Define distribution function of a random variable and give it's properties.

23. Let X be a random variable with probability density function p(x) and let g(X) be a real-valued function of X. Then prove that the expected value of g(X) is given by $E[g(X)] = \Sigma g(x)p(x)$.

24. The joint probability density function of (X, Y) is given by

$F(x, y) = 2$, for $0 \leq y \leq x \leq 1$.

$= 0$, elsewhere.

Check whether X and Y are independent.

Answer **any 6** short essay questions out of 9 : **(6x5=30)**

25. Find the moment generating function of a Poisson distribution and hence find the mean and variance.

26. Let X be a random variable following geometric distribution with parameter p, prove that $E(X) = 1/p$ and $V(X) = (1-p)/p^2$.

27. Show Poisson distribution as a limiting form of Binomial distribution.

28. Let Y be a continuous random variable with probability density function given by

$f(y) = 3y^2$, $0 \leq y \leq 1$

$= 0$, elsewhere.

Find F(y), $P(0.25 \leq Y \leq .75)$, graph both f(y) and F(y).

29. Prove the memoryless property of exponential distribution.

30 Find the mean and variance a beta distribution with parameters $\alpha > 0$ and $\beta > 0$.

31. If X is a normal variate with mean μ and standard deviation σ then find the moment generating function of $Y = (X - \mu)/\sigma$ and hence find the distribution of Y.