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- 32. Define probability generating function. Find the probability generating function of a binomial random variable and use it to find the mean.
- 33. Let X and Y be two independent random variables then prove that E(XY) = E(X)E(Y).

Answer any one essay question out of 2:

(1×10=10)

- 34. State and prove Tchebysheff's theorem and give the applications of it.
- 35. The length of time to failure (in hundreds of hours) for a transistor is a random variable X with distribution function given by

$$F(y) = 0,$$
 $y < 0$
= 1 - exp(-y²), $y \ge 0$.

- i) Show that F(x) has the properties of a distribution function.
- ii) Find f(x).
- iii) Find the probability that the transistor operates for at least 200 hours.
- iv) Find $P(X > 100/X \le 200)$.

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I Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.)

Examination, November 2016

BHM 105 : PROBABILITY THEORY – I

Time: 3 Hours Max. Marks: 80

Answer all the 10 questions :

(10×1=10)

- 1. Define a random variable.
- 2. Define expected value of a continuous random variable.
- 3. Define moment generating function of a random variable.
- 4. Define Poisson distribution.
- 5. Define variance of a random variable.
- If the mean and variance of Binomial distribution are 6 and 2 respectively, find the probability density function.
- 7. Define Gamma distribution.
- 8. Define the rth moment of a random variable about it's mean.
- 9. If $M_X(t)$ be the moment generating function of random variable X, find the moment generating function of Y = aX+ b.
- 10. If X and Y are two independent random variables, then what is the co-variance between X and Y?

Answer any 10 short answer questions:

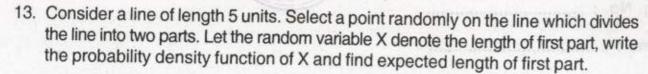
(10×3=30)

- 11. Find the moment generating function of a Binomial distribution.
- 12. If X has a geometric distribution with probability of success p, show that P(X is an odd number) = 1/(1 + q).

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- 14. The random variable X has a Poisson distribution and is such that P(0) = P(1). Find P(2).
- 15. If the distribution function of a random variable Y is given by

F(y) = 0, if y
$$\leq$$
 0
= y/8 if 0 < y < 2
= y²/16 if 2 \leq y < 4
= 1 if y \geq 4

Find:

- i) The density function of Y
- ii) $P(1 \le Y \le 3)$
- iii) $P(Y \ge 1/Y \le 3)$
- 16. Let X be a continuous random variable with density function f(x), where $x \ge 0$. If F(x) is the distribution function of X show that

$$E(X) = \int_{0}^{\infty} [1 - F(X)] dx$$

- 17. If X is a normal variate with mean 30 and variance 25, find
 - i) $P(X \ge 40)$
 - ii) P(X < 25)
 - iii) P(| X | < 35)
- 18. Find the moment generating function of a Gamma distribution and hence find the mean.
- 19. A random variable X has a uniform distribution over the interval (a, b). Derive the variance of Y.



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- 20. Explain how moments can be derived from moment generating function.
- 21. Let X be a random variable with mean 10 and variance 3 find a lower bound to P(5 < X < 15).
- 22. Define distribution function of a random variable and give it's properties.
- 23. Let X be a random variable with probability density function p(x) and let g(X) be a real-valued function of X. Then prove that the expected value of g(X) is given by E[g(X)] = Σg(x)p(x).
- 24. The joint probability density function of (X, Y) is given by

$$F(x, y) = 2$$
, for $0 \le y \le x \le 1$.

= 0, elsewhere.

Check whether X and Y are independent.

Answer any 6 short essay questions out of 9:

(6×5=30)

- 25. Find the moment generating function of a Poisson distribution and hence find the mean and variance.
- 26. Let X be a random variable following geometric distribution with parameter p, prove that E(X) = 1/p and $V(X) = (1 p)/p^2$.
- 27. Show Poisson distribution as a limiting form of Binomial distribution.
- 28. Let Y be a continuous random variable with probability density function given by

$$f(y) = 3y^2, 0 \le y \le 1$$

= 0, elsewhere.

Find F(y), $P(0.25 \le Y \le .75)$, graph both f(y) and F(y).

- 29. Prove the memoryless property of exponential distribution.
- 30 Find the mean and variance a beta distribution with parameters $\alpha > 0$ and $\beta > 0$.
- 31. If X is a normal variate with mean μ and standard deviation σ then find the moment generating function of Y = $(X \mu)/\sigma$ and hence find the distribution of Y.