26. Obtain the moment generating function of a random variable X having the density function $f(x)=\begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$.

Also find the first four moments about the origin.

SECTION - D

Answer any **2** questions out of **4** questions. Each question carries **6** marks. **(2×6=12)**

27. What do you mean by row equivalent matrices and row space of a matrix? Suppose that A and B are row equivalents matrices. Prove that the row space of A equals the row space of B.

28. Diagonalize the matrix $\begin{bmatrix} 2 & 0 & 0 \\ -3 & 4 & 1 \\ 3 & -2 & 1 \end{bmatrix}$.

29. The two dimensional random variable (X, Y) has the joint density function $f(x,y)=\frac{x+2y}{27}, x=0,1,2;\ y=0,1,2$. Find the conditional distribution of Y for $X = x$. Also find the conditional distribution of X given $Y = 1$.

30. Define moment generating function of a random variable. Write any two properties of moment generating function. If the probability density function of the random variable X has the probability law $P(x)=\frac{1}{20}e^{-\left|\frac{x-\theta}{\theta}\right|}$, $-\infty < x < \infty$, find the moment generating function of X, $E(X)$ and $\text{Var}(X)$.

Reg. No. : ....

Name : ....

I Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, October - 2019

(2016 Admission Onwards)

BHM-104 : MATRICES AND PROBABILITY THEORY

Time : 3 Hours Max. Marks : 60

SECTION - A

Answer any **4** questions out of **5** questions. Each question carries **1** mark. **(4×1=4)**

1. What is the length of the vector a = [2, 3, –1, 1]?

2. Give an example for a singular matrix.

3. What do you mean by continuous random variables.

4. What is the mean of a discrete random variable in terms of Mathematical expectation?

5. Define kurtosis of a distribution.

SECTION - B

Answer any **6** questions out of **9** questions. Each question carries **2** marks. **(6×2=12)**

6. Find the projection of x = [4, 0, –3] onto y = [3, 1, –7].

7. Explain Gauss elimination method to solve a system of linear equations.

8. Define rank of a matrix. What is the rank of the matrix $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix}$.

9. Find the determination of $A = \begin{bmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & -1 & -2 \end{bmatrix}$.

10. Define characteristic polynomial, eigen values and eigen vectors of a square matrix.

11. A fair coin is tossed 4 times. Find the probability distribution of the number of heads.

12. Find the value of E (X^2) of the random variable with probability density function $f(x) = ae^{-|x|}$, $-\infty \leq x \leq \infty$.

13. The number of components manufactured in a factory during a one month period is a random variable with mean 600 and variance 100. What is the probability that the production will be between 500 and 700 over a month?

14. Write a note on skewness of a distribution?

SECTION - C

Answer any **8** questions out of **12** questions. Each question carries **4** marks.

(8×4=32)

15. Using Gauss Jordan method, solve the system of linear equations:

$5x - 5y - 15z = 40$, $4x - 2y - 6z = 19$, $3x - 6y - 17z = 41$.

16. Determine whether the vector [5, 17, – 20] is in the row space of the matrix

$$\begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}.$$

17. Using row reduction method, find the inverse of matrix $\begin{bmatrix} -7 & 5 & 3 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \end{bmatrix}$.

18. Using row reduction, find the determinant of the matrix

$$\begin{bmatrix} 1 & -1 & 5 & 1 \\ -2 & 1 & -7 & 1 \\ -3 & 2 & -12 & -2 \\ 2 & -1 & 9 & 1 \end{bmatrix}.$$

19. Briefly explain the method for diagonalizing an $n \times n$ matrix.

20. What do you mean by algebraic multiplicity of an eigen value? Find the eigen values and their multiplicities of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & -5 \end{bmatrix}$.

21. Find the constant k so that the function $f(x) = \begin{cases} \frac{1}{k} & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$ is a density function, Also find the cumulative distribution function of the random variable X.

22. What do you mean by independent random variables? If

$$f(x, y) = \begin{cases} \frac{1}{4}(1 + xy), & |x| < 1, |y| < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

is the joint probability density functions of X and Y, show that X and Y are independent.

23. If X is a random variable and k is a real number, then prove that $Var[kX] = k^2 Var[X]$ and $Var[X + k] = Var[X]$.

24. The first four central moments of a distribution are 0, 2.5, 0.7 and 18.75. Test the skewness and kurtosis of the distribution.

25. The first four moments of a distribution about x = 4 are -1.5, 17, -30 and 108 respectively. Find the values the first four central moments.

P.T.O.